Math 110 | **BASIC CALCULUS I** | Fall 2007

Class 21

Wednesday October 24

Implicit Differentiation

Definition: implicit function

An equation in x and y variables of the form $F(x, y) = 0$ is said to define the function f implicitly if the graph of $y = f(x)$ coincides with some portion of the graph of the equation $F(x, y) = 0$.

What would such an implicitly defined function look like? Are these functions rare?

EXAMPLE

Consider the equation $x^2 + y^2 = 1$. What does the graph of this equation look like? Is this an implicitly defined function?

Exercise

(a) Solve the equation $8x^3 + 2y^5 = 1$ for x in terms of y.

(b) Now solve the same equation for y in terms of x.

(c) Is x a function of y or is y a function of x?

We say the equation $8x^3 + 2y^5 = 1$ gives x *implicitly* as a function of ____, while the equation $x = (1/2)\sqrt[3]{1 - 2y^5}$ gives x ________________ as a function of y.

Similarly, we say the equation $8x^3 + 2y^5 = 1$ gives y *implicitly* as a function of ____, while the equation $y =$________________ gives y explicitly as a function of x

Interpreting Implicit Differentiation As Related Rates Of Change

To understand the MEANING of implicit differentiation in terms of rates of change, fill in the following blanks.

$$\frac{d}{dy}\left[y^3\right] =$$

So, **at y** $= 2$, the rate of change of y^3 is ____.
This means increasing y by 1 unit causes y^3 to increase by ____ units.
Now, suppose y is a function of x. And suppose $\frac{dy}{dx} = 5$.
This means increasing x by 1 unit causes y to increase by ________ units, which in turn causes y^3 to increase by ________ units.
Implicit differentiation says exactly the same thing:

$$\frac{d}{dx}\left[y^3\right] =$$

Note this is identical to central concept of the Chain Rule, i.e. when $u = f(y)$, and $y = g(x)$
$\frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx}$

EXAMPLE

(a) Can you solve the equation $x^2 + y^3 = 8 - x + xy^5$ for y in terms of x?

(b) When $x = 0$, $y =$
(c) Surprising fact: We can find the slope of the graph at $x = 0$! (as follows)
Implicitly differentiate the above equation with respect to x, i.e., apply $\frac{d}{dx}$ to both sides of the equation.

Now plug in $x = 0$ and $y =$____, and then solve for $\frac{dy}{dx}$.

Exercise

Find the equation of the tangent line to the graph of $x^2 + y^3 = 8 - x + xy^5$ at $x = 0$.

Extending The Power Rule To Rational Powers

Suppose $y = x^r$ where r is a rational number, i.e. $r = m/n$ where m and n are integers. We can use implicit differentiation to show that $(x^r)' = rx^{r-1}$. (HINT: $y = x^{m/n} \Leftrightarrow y^n = x^m$)