Basic Calculus I

Class 15

Friday October 5

The Derivative Function

EXAMPLE

1. Let $f(x) = x^2$. Find the following derivatives algebraically.

(a) Find $f'(5)$

(b) Find $f'(122)$

(c) Find $f'(a)$.

f' is a **function**:

input = 5, output = _____; input = 122, output = _____; input = a, output = _____.

So we write $f'(x) =$ _____.

Exercise

2. (a) Let $f(x) = 5x + 3$. Find its derivative $f'(x)$ algebraically.

(b) $f'(28) =$ ____; $f'(0) =$ ____; $f'(-8) =$ ____.

Does this make sense? (Think about the graph of f and its slope.)

DEFINITION: The Derivative Function f'

The function $f'(x)$ defined by by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

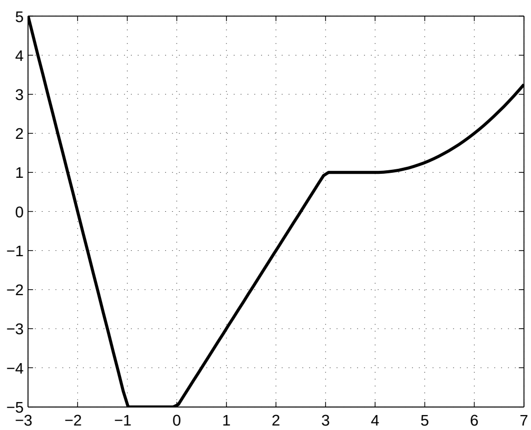
is called **the derivative of** f **with respect to** x. The domain of the function f' is the set of all x values in the domain of $f(x)$ for which the limit of the difference quotient (shown above) exists.

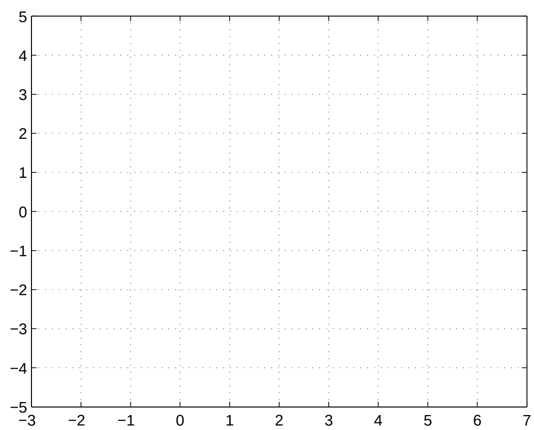
EXAMPLE

3. Differentiate $f(x) = 1/x$ algebraically.

GROUPWORK

4. (a) Suppose the graph of a function $g(x)$ is as shown below. Sketch a graph of its derivative $g'(x)$ on the empty axes to the right.





(b) On which intervals is g increasing?
What do you notice about g' on these intervals?

(c) On which intervals is g decreasing?
What do you notice about g' on these intervals?

(d) On which intervals is g constant?
What do you notice about g' on these intervals?

DEFINITION: differentiability

A function f is said to be differentiable at the point x_0 if the limit

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

exists. If a function is differentiable at every point on the open interval (a, b) then we say that the function f is **differentiable on** (a, b).