Math 110 **BASIC CALCULUS I** Fall 2007

Class 10

Friday September 21

Infinite Limits and Limits at Infinity

DEFINITION: **Limits At Infinity**

An informal definition of **the limit of a function** $f(x)$ **as** x **increases without bound** is if by taking (input) values of x larger and larger the (output) values of $f(x)$ can be made as close to a number L as desired. Symbolically, this is denoted

$$\lim_{x\to\infty} f(x) = L \quad \text{or} \quad f(x) \to L \text{ as } x \to \infty$$

Graphically, this means that the graph of $y = f(x)$ has a **horizontal asymptote** at $y = L$. In fact, someetime the behavior of a function as x "goes to" $\pm\infty$ is called the "end behavior" or **asymptotic** behavior of the function.

CLICKER QUESTION

What is the maximum number of **horizontal** asymptotes that the graph of a function can have?

(a) One

(b) Two

(c) Three

(d) As Many As We Want

CLICKER QUESTION

What is the maximum number of **vertical** asymptotes that the graph of a function can have?

(a) One

(b) Two

(c) Three

(d) As Many As We Want

DEFINITION: **Infinite Limits**

If the output values of $f(x)$ increase without bound (either positively or negatively) as the input values are increased without bound (either positively or negatively) we say that $f(x)$ "goes to infinity" as x "goes to infinity" and this can be written as

$$\lim_{x\to\infty} f(x) = +\infty \quad \text{or} \quad f(x) \to +\infty \text{ as } x \to \infty$$

or

$$\lim_{x\to-\infty} f(x) = +\infty \quad \text{or} \quad f(x) \to +\infty \text{ as } x \to -\infty$$

or

$$\lim_{x\to\infty} f(x) = -\infty \quad \text{or} \quad f(x) \to -\infty \text{ as } x \to \infty$$

or

$$\lim_{x\to-\infty} f(x) = -\infty \quad \text{or} \quad f(x) \to -\infty \text{ as } x \to -\infty$$

In order to regularly compute limits with infinity we need to be familiar with how to compute some standard infinite limits

THEOREM

Here are some standard limits involving infinity

(a) $\lim\limits_{x\to\infty} \frac{1}{x} = 0$

(b) $\lim\limits_{x\to-\infty} \frac{1}{x} = 0$

(c) $\lim\limits_{x\to\infty} C = C$, for any constant C

(d) $\lim\limits_{x\to-\infty} C = C$, for any constant C

(e) $\lim\limits_{x\to\infty} b^x = \infty$ for any $b > 0$

(f) $\lim\limits_{x\to-\infty} b^x = 0$ for any $b > 0$

(g) $\lim\limits_{x\to\infty} \log_b(x) = +\infty$ for any $b > 0$

(h) $\lim\limits_{x\to 0^+} \log_b(x) = -\infty$ for any $b > 0$

(i) $\lim\limits_{x\to\infty} \sin(x) =$ DOES NOT EXIST

(j) $\lim\limits_{x\to-\infty} \sin(x) =$ DOES NOT EXIST

GroupWork

Draw a picture representing each one of the basic limits given above, in the space below.

Infinite Limits Of Polynomials

Again, polynomials reveal their tractable nature:

$\lim_{x\to\infty} p(x) = \lim_{x\to\infty} c_n x^n = +\infty$ or $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty} c_n x^n$

where $p(x) = c + 0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots + c_n x^n$. In other words to evaluate an infinite limit with a polynomial function, all one has to do is look at the term of the polynomial with the

____.

Exercise

Evaluate $\lim_{x\to\infty} x^3 + 3x - 4$ and $\lim_{x\to-\infty} x^2 - x - 1$

Infinite Limits of Rational Functions

As usual, Rational Functions require some careful attention to deal with. However, we use our intuition that rational functions generally behave as a ratio of two polynomials. In that case, in order to evaluate the end behavior of a rational function depends on the highest degree term in the numerator divided by the highest degree term in the denominator.

In other words,

$$\lim_{x\to\infty} \frac{p(x)}{q(x)} = \lim_{x\to\infty} \frac{c_0 + c_1 x + c_2 x^2 + \ldots c_n x^n}{d_0 + d_1 x + d_2 x^2 + \ldots d_n x^n} = \lim_{x\to\infty} \frac{c_n x^n}{d_n x^n}$$

EXAMPLES

Anton, Bivens & Davis, **page 131, #12, 15, 17**

Evaluate the following limits

$$\lim_{x\to\infty} \frac{5x^2 - 4x}{2x^2 + 3}$$

$$\lim_{x\to-\infty} \frac{x - 2}{x^2 + 2x + 1}$$

$$\lim_{x\to\infty} \sqrt[3]{\frac{2 + 3x - 5x^2}{1 + 8x^2}}$$

CLICKER QUESTION

If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$ then $\lim_{x\to a}[f(x) - g(x)] =$

(a) 0

(b) ∞

(c) Does Not Exist

(d) Not enough information given to evaluate the limit

CLICKER QUESTION

TRUE or FALSE: At some point in time, you were once exactly 3 feet tall.

(a) TRUE

(b) FALSE