Math 110

Basic Calculus I

Class 9

Wednesday September 19

Computing Limits

Fall 2007

Basic Limits and Algebraic Rules

In order to compute limits we need tobe familiar with how to compute some basic limits as well as apply some typical rules

THEOREM

Here are some basic limits

(a) $\lim_{x \to a} C = C$, for any constant C

(b) $\lim_{x \to a} x = a$

(c) $\lim_{x \to 0^-} x = -\infty$

(d) $\lim_{x \to 0^+} x = +\infty$

(e) $\lim_{x \to a^+} \frac{1}{x-a} = +\infty$

(f) $\lim_{x \to a^-} \frac{1}{x-a} = -\infty$

(g) $\lim_{x \to a} \frac{1}{(x-a)^2} = \infty$

GroupWork

Draw a picture representing each one of the basic limits given above, in the space below.

THEOREM

Here are some algebraic rules involving limits.
Suppose that the limits $\lim_{x\to a} f(x) = L_1$ and $\lim_{x\to a} g(x) = L_2$ exist. Then

(a) The limit of a difference is the difference of the limits:
$\lim_{x\to a}(f(x) - g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = L_1 - L_2$

(b) The limit of a sum is the sum of the limits:
$\lim_{x\to a}(f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = L_1 + L_2$

(c) The limit of a multiple is a multiple of the limit:
$\lim_{x\to a} cf(x) = c \cdot \lim_{x\to a} f(x) = c \cdot L_1,$ where c is a constant

(d) The limit of a product is the product of the limits:
$\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = L_1 L_2$

(e) The limit of a quotient is the quotient of the limits, as long as the limit of the denominator is NOT zero:
$\lim_{x\to a} f(x)/g(x) = \lim_{x\to a} f(x)/ \lim_{x\to a} g(x) = L_1/L_2,$ provided $\lim_{x\to a} g(x) = L_2 \neq 0$

(f) The limit of an n^{th} root is the n^{th} root of a limit:
$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} = \sqrt[n]{L_1}$ as long as when n is even$L_1 > 0$

All of the above rules apply for one sided-limits also.

Exercise

Anton, Bivens & Davis **, page 121, #1**

Suppose $\lim_{x\to a} f(x) = 2,$ $\lim_{x\to a} g(x) = -4$ and $\lim_{x\to a} h(x) = 0$
Find the limits that exist. If the limt does not exist, explain why.

(a) $\lim_{x\to a}[h(x) - 3g(x) + 1]$

(b) $\lim_{x\to a}[g(x)]^2$

(c) $\lim_{x\to a}[\sqrt[3]{6 + f(x)}]$

(d) $\lim_{x\to a}[\frac{3f(x) - 8g(x)}{h(x)}$

(e) $\lim_{x\to a}[\frac{7g(x)}{2f(x) + g(x)}$

Limits of Polynomials and Rational Functions

$\lim_{x \to a} p(x) = p(a)$ where p(x) is a polynomial function of the form $c_0 + c_1 x + c_2 x^2 + \ldots c_n x^n$

Limit of Rational Functions are a bit more interesting. Recall, a Rational Function is a ratio of two polynomials.

$$\lim_{x \to a} \frac{p(x)}{q(x)} = \begin{cases} \frac{p(a)}{q(a)}, & \text{if } q(a) \neq 0 \\ \text{DOES NOT EXIST}, & \text{if } q(a) = 0 \text{ and } p(a) \neq 0 \end{cases}$$

Indeterminate Form

When $q(a) = 0$ and $p(a) = 0$ the limit is said to be "an indeterminate form of the form 0/0." This limit can have **any value** or it may also not exist. Typically, what one tries to do is use some kind of algebraic simplification tin order to evaluate the limit. Later on, we will learn a technique called **L'Hôpital's Rule** which will allow us to evaluate indeterminate forms precisely.

Exercise Evaluate the following limits

(a) $\lim_{x \to 4} \frac{2x + 8}{x^2 + x - 12}$

(b) $\lim_{x \to 5} \frac{x^2 - 3x - 10}{x^2 - 10x + 25}$

EXAMPLE

One can have indeterminate forms with functions other than rational functions also. Evaluate $\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1}$.

CLICKER QUESTION

$$f(x) = \begin{cases} x+1, & \text{if } x \leq 1 \\ x-1, & \text{if } x > 1 \end{cases}$$

Which of the following limits does not exist?

(a) $\lim_{x \to 1^-} f(x)$

(b) $\lim_{x \to 1^+} f(x)$

(c) $\lim_{x \to 1} f(x)$

(d) All of the above.