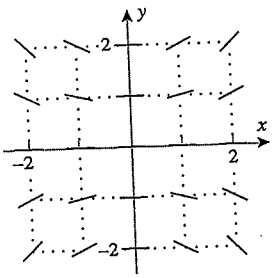
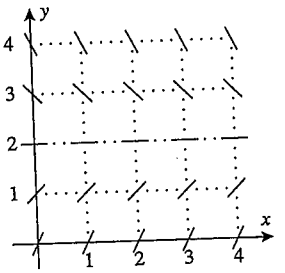
EXERCISE SET 9.2

1. $y^1 = xy/4$
$-2 \le x \le +2$
$-2 \le y \le +2$



2.



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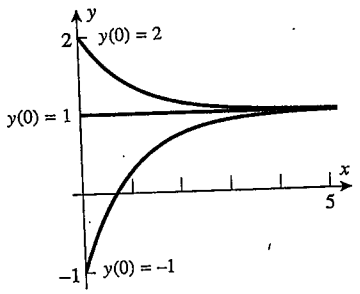


4. $\dfrac{dy}{dx} + y = 1,\ \mu = e^{\int dx} = e^x,$

$\dfrac{d}{dx}[ye^x] = e^x,$

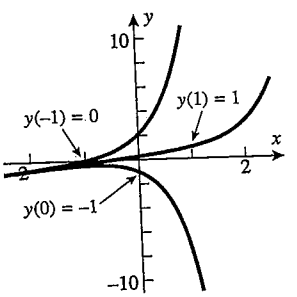
$ye^x = e^x + C, y = 1 + Ce^{-x}$

(a) $-1 = 1 + C, C = -2, y = 1 - 2e^{-x}$

(b) $1 = 1 + C, C = 0, y = 1$

(c) $2 = 1 + C, C = 1, y = 1 + e^{-x}$

5.



Exercise Set 9.2

6. $\dfrac{dy}{dx} - 2y = -x, \quad \mu = e^{-2\int dx} = e^{-2x}, \quad \dfrac{d}{dx}\left[ye^{-2x}\right] = -xe^{-2x},$

$ye^{-2x} = \dfrac{1}{4}(2x+1)e^{-2x} + C, \quad y = \dfrac{1}{4}(2x+1) + Ce^{2x}$

(a) $1 = 3/4 + Ce^2$, $C = 1/(4e^2)$, $y = \dfrac{1}{4}(2x+1) + \dfrac{1}{4}e^{2x-2}$

(b) $-1 = 1/4 + C, C = -5/4, y = \dfrac{1}{4}(2x+1) - \dfrac{5}{4}e^{2x}$

(c) $0 = -1/4 + Ce^{-2}$, $C = e^2/4$, $y = \dfrac{1}{4}(2x+1) + \dfrac{1}{4}e^{2x+2}$

7. $\lim\limits_{x\to+\infty} y = 1$

8. $\lim\limits_{x\to+\infty} y = \begin{cases} +\infty & \text{if } y_0 \geq 1/4 \\ -\infty, & \text{if } y_0 < 1/4 \end{cases}$

9. (a) IV, since the slope is positive for $x > 0$ and negative for $x < 0$.

(b) VI, since the slope is positive for $y > 0$ and negative for $y < 0$.

(c) V, since the slope is always positive.

(d) II, since the slope changes sign when crossing the lines $y = \pm 1$.

(e) I, since the slope can be positive or negative in each quadrant but is not periodic.

(f) III, since the slope is periodic in both x and y.

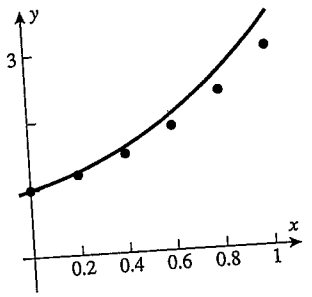
11. (a) $y_0 = 1$,
$y_{n+1} = y_n + (x_n + y_n)(0.2) = (x_n + 6y_n)/5$

n	0	1	2	3	4	5
x_n	0	0.2	0.4	0.6	0.8	1.0
y_n	1	1.20	1.48	1.86	2.35	2.98

(b) $y' - y = x$, $\mu = e^{-x}$, $\dfrac{d}{dx}\left[ye^{-x}\right] = xe^{-x}$,
$ye^{-x} = -(x+1)e^{-x} + C$, $1 = -1 + C$,
$C = 2$, $y = -(x+1) + 2e^x$

x_n	0	0.2	0.4	0.6	0.8	1.0
$y(x_n)$	1	1.24	1.58	2.04	2.65	3.44
abs. error	0	0.04	0.10	0.19	0.30	0.46
perc. error	0	3	6	9	11	13

(c)



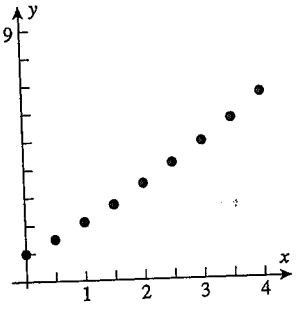
12. $h = 0.1, y_{n+1} = (x_n + 11y_n)/10$

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_n	1.00	1.10	1.22	1.36	1.53	1.72	1.94	2.20	2.49	2.82	3.19

In Exercise 11, $y(1) \approx 2.98$; in Exercise 12, $y(1) \approx 3.19$; the true solution is $y(1) \approx 3.44$; so the absolute errors are approximately 0.46 and 0.25 respectively.

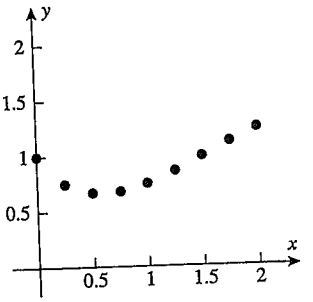
13. $y_0 = 1, y_{n+1} = y_n + \frac{1}{2}y_n^{1/3}$

n	0	1	2	3	4	5	6	7	8
x_n	0	0.5	1	1.5	2	2.5	3	3.5	4
y_n	1	1.50	2.11	2.84	3.68	4.64	5.72	6.91	8.23



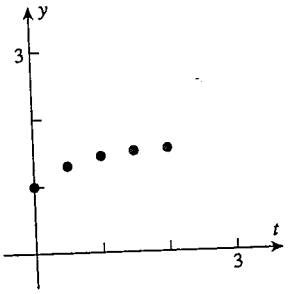
14. $y_0 = 1, y_{n+1} = y_n + (x_n - y_n^2)/4$

n	0	1	2	3	4	5	6	7	8
x_n	0	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
y_n	1	0.75	0.67	0.68	0.75	0.86	0.99	1.12	1.24



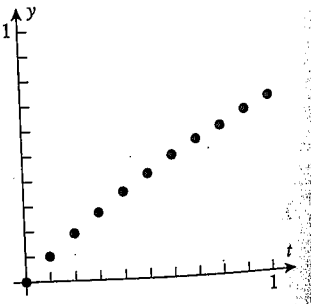
15. $y_0 = 1, y_{n+1} = y_n + \dfrac{1}{2}\cos y_n$

n	0	1	2	3	4
t_n	0	0.5	1	1.5	2
y_n	1	1.27	1.42	1.49	1.53



16. $y_0 = 0,\ y_{n+1} = y_n + e^{-y_n}/10$

n	0	1	2	3	4	5	6	7	8	9	10
t_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_n	0	0.10	0.19	0.27	0.35	0.42	0.49	0.55	0.60	0.66	0.71



17. $h = 1/5,\ y_0 = 1,\ y_{n+1} = y_n + \dfrac{1}{5}\sin(\pi n/5)$

n	0	1	2	3	4	5
t_n	0	0.2	0.4	0.6	0.8	1.0
y_n	1	1	1.12	1.31	1.50	1.62