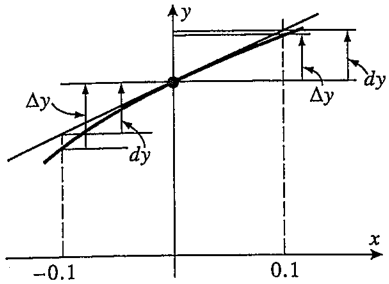
EXERCISE SET 3.8

1. (a) $f(x) \approx f(1) + f'(1)(x-1) = 1 + 3(x-1)$
   (b) $f(1+\Delta x) \approx f(1) + f'(1)\Delta x = 1 + 3\Delta x$
   (c) From Part (a), $(1.02)^3 \approx 1 + 3(0.02) = 1.06$. From Part (b), $(1.02)^3 \approx 1 + 3(0.02) = 1.06$.

2. (a) $f(x) \approx f(2) + f'(2)(x-2) = 1/2 + (-1/2^2)(x-2) = (1/2) - (1/4)(x-2)$
   (b) $f(2+\Delta x) \approx f(2) + f'(2)\Delta x = 1/2 - (1/4)\Delta x$
   (c) From Part (a), $1/2.05 \approx 0.5 - 0.25(0.05) = 0.4875$, and from Part (b),
   $1/2.05 \approx 0.5 - 0.25(0.05) = 0.4875$.

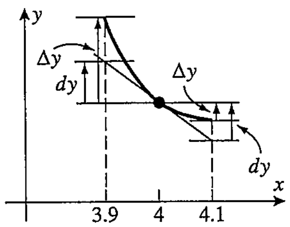
3. (a) $f(x) \approx f(x_0) + f'(x_0)(x - x_0) = 1 + (1/(2\sqrt{1})(x - 0) = 1 + (1/2)x$, so with $x_0 = 0$ and $x = -0.1$, we have $\sqrt{0.9} = f(-0.1) \approx 1 + (1/2)(-0.1) = 1 - 0.05 = 0.95$. With $x = 0.1$ we have $\sqrt{1.1} = f(0.1) \approx 1 + (1/2)(0.1) = 1.05$.

(b)



4. (a) $f(x) \approx f(x_0)+f'(x_0)(x-x_0) = 1/2-\left[1/(2\cdot 4^{3/2})\right](x-4) = 1/2-(x-4)/16$, so with $x_0 = 4$ and $x = 3.9$ we have $1/\sqrt{3.9} = f(3.9) \approx 0.5 - (-0.1)/16 = 0.50625$. If $x_0 = 4$ and $x = 4.1$ then $1/\sqrt{4.1} = f(4.1) \approx 0.5 - (0.1)/16 = 0.49375$

(b)



5. $f(x) = (1+x)^{15}$ and $x_0 = 0$. Thus $(1+x)^{15} \approx f(x_0)+f'(x_0)(x-x_0) = 1+15(1)^{14}(x-0) = 1+15x$.

6. $f(x) = \dfrac{1}{\sqrt{1-x}}$ and $x_0 = 0$, so $\dfrac{1}{\sqrt{1-x}} \approx f(x_0)+f'(x_0)(x-x_0) = 1+\dfrac{1}{2(1-0)^{3/2}}(x-0) = 1+x/2$

7. $\tan x \approx \tan(0) + \sec^2(0)(x - 0) = x$

8. $\dfrac{1}{1+x} \approx 1 + \dfrac{-1}{(1+0)^2}(x - 0) = 1 - x$

9. $x^4 \approx (1)^4 + 4(1)^3(x - 1)$. Set $\Delta x = x - 1$; then $x = \Delta x + 1$ and $(1 + \Delta x)^4 = 1 + 4\Delta x$.

10. $\sqrt{x} \approx \sqrt{1} + \dfrac{1}{2\sqrt{1}}(x - 1)$, and $x = 1 + \Delta x$, so $\sqrt{1 + \Delta x} \approx 1 + \Delta x/2$

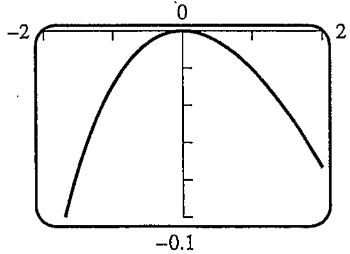
11. $\dfrac{1}{2+x} \approx \dfrac{1}{2+1} - \dfrac{1}{(2+1)^2}(x-1)$, and $2 + x = 3 + \Delta x$, so $\dfrac{1}{3+\Delta x} \approx \dfrac{1}{3} - \dfrac{1}{9}\Delta x$

12. $(4 + x)^3 \approx (4 + 1)^3 + 3(4 + 1)^2(x - 1)$ so, with $4 + x = 5 + \Delta x$ we get $(5 + \Delta x)^3 \approx 125 + 75\Delta x$

13. $f(x) = \sqrt{x+3}$ and $x_0 = 0$, so

$\sqrt{x+3} \approx \sqrt{3} + \dfrac{1}{2\sqrt{3}}(x - 0) = \sqrt{3} + \dfrac{1}{2\sqrt{3}}x$, and

$\left|f(x) - \left(\sqrt{3} + \dfrac{1}{2\sqrt{3}}x\right)\right| < 0.1$ if $|x| < 1.692$.



$\left|f(x) - \left(\sqrt{3} + \frac{1}{2\sqrt{3}}x\right)\right|$

19. $f(x) = x^4$, $f'(x) = 4x^3$, $x_0 = 3$, $\Delta x = 0.02$; $(3.02)^4 \approx 3^4 + (108)(0.02) = 81 + 2.16 = 83.16$

20. $f(x) = x^3$, $f'(x) = 3x^2$, $x_0 = 2$, $\Delta x = -0.03$; $(1.97)^3 \approx 2^3 + (12)(-0.03) = 8 - 0.36 = 7.64$

21. $f(x) = \sqrt{x}$, $f'(x) = \dfrac{1}{2\sqrt{x}}$, $x_0 = 64$, $\Delta x = 1$; $\sqrt{65} \approx \sqrt{64} + \dfrac{1}{16}(1) = 8 + \dfrac{1}{16} = 8.0625$

22. $f(x) = \sqrt{x}$, $f'(x) = \dfrac{1}{2\sqrt{x}}$, $x_0 = 25$, $\Delta x = -1$; $\sqrt{24} \approx \sqrt{25} + \dfrac{1}{10}(-1) = 5 - 0.1 = 4.9$

23. $f(x) = \sqrt{x}$, $f'(x) = \dfrac{1}{2\sqrt{x}}$, $x_0 = 81$, $\Delta x = -0.1$; $\sqrt{80.9} \approx \sqrt{81} + \dfrac{1}{18}(-0.1) \approx 8.9944$

24. $f(x) = \sqrt{x}$, $f'(x) = \dfrac{1}{2\sqrt{x}}$, $x_0 = 36$, $\Delta x = 0.03$; $\sqrt{36.03} \approx \sqrt{36} + \dfrac{1}{12}(0.03) = 6 + 0.0025 = 6.0025$

25. $f(x) = \sin x$, $f'(x) = \cos x$, $x_0 = 0$, $\Delta x = 0.1$; $\sin 0.1 \approx \sin 0 + (\cos 0)(0.1) = 0.1$

26. $f(x) = \tan x$, $f'(x) = \sec^2 x$, $x_0 = 0$, $\Delta x = 0.2$; $\tan 0.2 \approx \tan 0 + (\sec^2 0)(0.2) = 0.2$

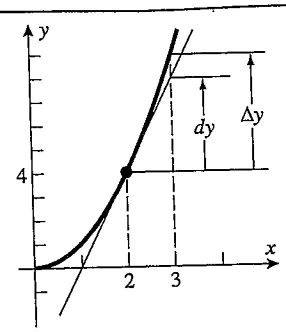
27. $f(x) = \cos x$, $f'(x) = -\sin x$, $x_0 = \pi/6$, $\Delta x = \pi/180$;

$$\cos 31^\circ \approx \cos 30^\circ + \left(-\frac{1}{2}\right)\left(\frac{\pi}{180}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{360} \approx 0.8573$$

28. (a) Let $f(x) = (1+x)^k$ and $x_0 = 0$. Then $(1+x)^k \approx 1^k + k(1)^{k-1}(x-0) = 1 + kx$. Set $k = 37$ and $x = 0.001$ to obtain $(1.001)^{37} \approx 1.037$.
(b) With a calculator $(1.001)^{37} = 1.03767$.
(c) The approximation is $(1.1)^{37} \approx 1 + 37(0.1) = 4.7$, and the calculator value is 34.004. The error is due to the relative largeness of $f'(1)\Delta x = 37(0.1) = 3.7$.

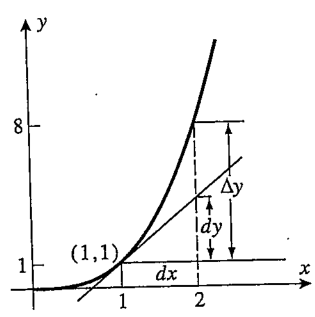
29. (a) $dy = f'(x)dx = 2xdx = 4(1) = 4$ and $\Delta y = (x + \Delta x)^2 - x^2 = (2+1)^2 - 2^2 = 5$

(b)



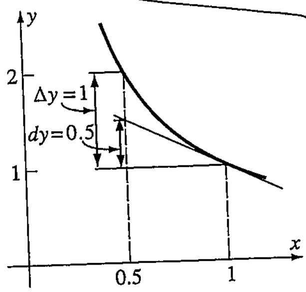
30. (a) $dy = 3x^2 dx = 3(1)^2(1) = 3$ and $\Delta y = (x + \Delta x)^3 - x^3 = (1+1)^3 - 1^3 = 7$

(b)



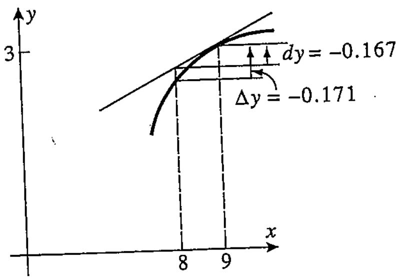
31. (a) $dy = (-1/x^2)dx = (-1)(-0.5) = 0.5$ and
$\Delta y = 1/(x + \Delta x) - 1/x$
$= 1/(1 - 0.5) - 1/1 = 2 - 1 = 1$

(b)



32. (a) $dy = (1/2\sqrt{x})dx = (1/(2 \cdot 3))(-1) = -1/6 \approx -0.167$ and
$\Delta y = \sqrt{x + \Delta x} - \sqrt{x} = \sqrt{9 + (-1)} - \sqrt{9} = \sqrt{8} - 3 \approx -0.172$

(b)



33. $dy = 3x^2 dx$;
$\Delta y = (x + \Delta x)^3 - x^3 = x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3 = 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3$

34. $dy = 8dx$; $\Delta y = [8(x + \Delta x) - 4] - [8x - 4] = 8\Delta x$

35. $dy = (2x - 2)dx$;
$\Delta y = [(x + \Delta x)^2 - 2(x + \Delta x) + 1] - [x^2 - 2x + 1]$
$= x^2 + 2x\ \Delta x + (\Delta x)^2 - 2x - 2\Delta x + 1 - x^2 + 2x - 1 = 2x\ \Delta x + (\Delta x)^2 - 2\Delta x$

36. $dy = \cos x\, dx$; $\Delta y = \sin(x + \Delta x) - \sin x$

37. (a) $dy = (12x^2 - 14x)dx$
(b) $dy = x\, d(\cos x) + \cos x\, dx = x(-\sin x)dx + \cos x dx = (-x \sin x + \cos x)dx$

38. (a) $dy = (-1/x^2)dx$ (b) $dy = 5 \sec^2 x\, dx$

39. (a) $dy = \left(\sqrt{1 - x} - \dfrac{x}{2\sqrt{1 - x}}\right) dx = \dfrac{2 - 3x}{2\sqrt{1 - x}} dx$

(b) $dy = -17(1 + x)^{-18} dx$

40. (a) $dy = \dfrac{(x^3 - 1)d(1) - (1)d(x^3 - 1)}{(x^3 - 1)^2} = \dfrac{(x^3 - 1)(0) - (1)3x^2 dx}{(x^3 - 1)^2} = -\dfrac{3x^2}{(x^3 - 1)^2} dx$

(b) $dy = \dfrac{(2 - x)(-3x^2)dx - (1 - x^3)(-1)dx}{(2 - x)^2} = \dfrac{2x^3 - 6x^2 + 1}{(2 - x)^2} dx$

41. $dy = \dfrac{3}{2\sqrt{3x - 2}} dx$, $x = 2$, $dx = 0.03$; $\Delta y \approx dy = \dfrac{3}{4}(0.03) = 0.0225$

42. $dy = \dfrac{x}{\sqrt{x^2 + 8}} dx$, $x = 1$, $dx = -0.03$; $\Delta y \approx dy = (1/3)(-0.03) = -0.01$

43. $dy = \dfrac{1-x^2}{(x^2+1)^2}dx$, $x = 2$, $dx = -0.04$; $\Delta y \approx dy = \left(-\dfrac{3}{25}\right)(-0.04) = 0.0048$

44. $dy = \left(\dfrac{4x}{\sqrt{8x+1}} + \sqrt{8x+1}\right)dx$, $x = 3$, $dx = 0.05$; $\Delta y \approx dy = (37/5)(0.05) = 0.37$

45. (a) $A = x^2$ where x is the length of a side; $dA = 2x\,dx = 2(10)(\pm 0.1) = \pm 2$ ft^2.

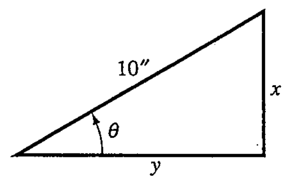
(b) relative error in x is $\approx \dfrac{dx}{x} = \dfrac{\pm 0.1}{10} = \pm 0.01$ so percentage error in x is $\approx \pm 1\%$; relative error in A is $\approx \dfrac{dA}{A} = \dfrac{2x\,dx}{x^2} = 2\dfrac{dx}{x} = 2(\pm 0.01) = \pm 0.02$ so percentage error in A is $\approx \pm 2\%$

46. (a) $V = x^3$ where x is the length of a side; $dV = 3x^2dx = 3(25)^2(\pm 1) = \pm 1875$ cm^3.

(b) relative error in x is $\approx \dfrac{dx}{x} = \dfrac{\pm 1}{25} = \pm 0.04$ so percentage error in x is $\approx \pm 4\%$; relative error in V is $\approx \dfrac{dV}{V} = \dfrac{3x^2dx}{x^3} = 3\dfrac{dx}{x} = 3(\pm 0.04) = \pm 0.12$ so percentage error in V is $\approx \pm 12\%$

47. (a) $x = 10\sin\theta$, $y = 10\cos\theta$ (see figure),

$$dx = 10\cos\theta d\theta = 10\left(\cos\frac{\pi}{6}\right)\left(\pm\frac{\pi}{180}\right) = 10\left(\frac{\sqrt{3}}{2}\right)\left(\pm\frac{\pi}{180}\right)$$
$$\approx \pm 0.151 \text{ in},$$
$$dy = -10(\sin\theta)d\theta = -10\left(\sin\frac{\pi}{6}\right)\left(\pm\frac{\pi}{180}\right) = -10\left(\frac{1}{2}\right)\left(\pm\frac{\pi}{180}\right)$$
$$\approx \pm 0.087 \text{ in}$$

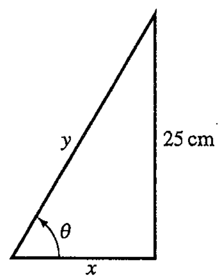


(b) relative error in x is $\approx \dfrac{dx}{x} = (\cot\theta)d\theta = \left(\cot\dfrac{\pi}{6}\right)\left(\pm\dfrac{\pi}{180}\right) = \sqrt{3}\left(\pm\dfrac{\pi}{180}\right) \approx \pm 0.030$ so percentage error in x is $\approx \pm 3.0\%$;

relative error in y is $\approx \dfrac{dy}{y} = -\tan\theta d\theta = -\left(\tan\dfrac{\pi}{6}\right)\left(\pm\dfrac{\pi}{180}\right) = -\dfrac{1}{\sqrt{3}}\left(\pm\dfrac{\pi}{180}\right) \approx \pm 0.010$ so percentage error in y is $\approx \pm 1.0\%$

48. (a) $x = 25\cot\theta$, $y = 25\csc\theta$ (see figure);

$$dx = -25\csc^2\theta d\theta = -25\left(\csc^2\frac{\pi}{3}\right)\left(\pm\frac{\pi}{360}\right)$$
$$= -25\left(\frac{4}{3}\right)\left(\pm\frac{\pi}{360}\right) \approx \pm 0.291 \text{ cm},$$
$$dy = -25\csc\theta\cot\theta d\theta = -25\left(\csc\frac{\pi}{3}\right)\left(\cot\frac{\pi}{3}\right)\left(\pm\frac{\pi}{360}\right)$$
$$= -25\left(\frac{2}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\left(\pm\frac{\pi}{360}\right) \approx \pm 0.145 \text{ cm}$$



(b) relative error in x is $\approx \dfrac{dx}{x} = -\dfrac{\csc^2\theta}{\cot\theta}d\theta = -\dfrac{4/3}{1/\sqrt{3}}\left(\pm\dfrac{\pi}{360}\right) \approx \pm 0.020$ so percentage error in x is $\approx \pm 2.0\%$; relative error in y is $\approx \dfrac{dy}{y} = -\cot\theta d\theta = -\dfrac{1}{\sqrt{3}}\left(\pm\dfrac{\pi}{360}\right) \approx \pm 0.005$ so percentage error in y is $\approx \pm 0.5\%$