35. (a) $2(1+x^{-1})(x^{-3}+7)+(2x+1)(-x^{-2})(x^{-3}+7)+(2x+1)(1+x^{-1})(-3x^{-4})$

(b) $(x^7+2x-3)^3=(x^7+2x-3)(x^7+2x-3)(x^7+2x-3)$ so

$$\begin{aligned}\frac{d}{dx}(x^7+2x-3)^3 &= (7x^6+2)(x^7+2x-3)(x^7+2x-3)\\ &\quad+(x^7+2x-3)(7x^6+2)(x^7+2x-3)\\ &\quad+(x^7+2x-3)(x^7+2x-3)(7x^6+2)\\ &=3(7x^6+2)(x^7+2x-3)^2\end{aligned}$$

36. (a) $-5x^{-6}(x^2+2x)(4-3x)(2x^9+1)+x^{-5}(2x+2)(4-3x)(2x^9+1)$
$+x^{-5}(x^2+2x)(-3)(2x^9+1)+x^{-5}(x^2+2x)(4-3x)(18x^8)$

(b) $(x^2+1)^{50}=(x^2+1)(x^2+1)\cdots(x^2+1)$, where (x^2+1) occurs 50 times so

$$\begin{aligned}\frac{d}{dx}(x^2+1)^{50} &= [(2x)(x^2+1)\cdots(x^2+1)]+[(x^2+1)(2x)\cdots(x^2+1)]\\ &\quad+\cdots+[(x^2+1)(x^2+1)\cdots(2x)]\\ &=2x(x^2+1)^{49}+2x(x^2+1)^{49}+\cdots+2x(x^2+1)^{49}\\ &=100x(x^2+1)^{49} \text{ because } 2x(x^2+1)^{49} \text{ occurs 50 times.}\end{aligned}$$

37. By the product rule, $g'(x)$ is the sum of n terms, each containing n factors of the form $f'(x)f(x)f(x)\dots f(x)$; the function $f(x)$ occurs $n-1$ times, and $f'(x)$ occurs once. Each of these n terms is equal to $f'(x)(f(x))^{n-1}$, and so $g'(x)=n(f(x))^{n-1}f'(x)$.

38. $g'(x)=10(x^2-1)^9(2x)=20x(x^2-1)^9$

39. $f(x)=\dfrac{1}{x^n}$ so $f'(x)=\dfrac{x^n\cdot(0)-1\cdot(nx^{n-1})}{x^{2n}}=-\dfrac{n}{x^{n+1}}$

40. $f(x)=g(x)h(x)$, $f'(x)=g'(x)h(x)+g(x)h'(x)$, solve for h': $h'(x)=\dfrac{f'(x)-g'(x)h(x)}{g(x)}$, but $h=f/g$ so $\dfrac{d}{dx}\dfrac{f(x)}{g(x)}=\dfrac{f'(x)}{g(x)}-\dfrac{f(x)g'(x)}{g(x)^2}=\dfrac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$

EXERCISE SET 3.5

1. $f'(x)=-4\sin x+2\cos x$

2. $f'(x)=\dfrac{-10}{x^3}+\cos x$

3. $f'(x)=4x^2\sin x-8x\cos x$

4. $f'(x)=4\sin x\cos x$

5. $f'(x)=\dfrac{\sin x(5+\sin x)-\cos x(5-\cos x)}{(5+\sin x)^2}=\dfrac{1+5(\sin x-\cos x)}{(5+\sin x)^2}$

6. $f'(x)=\dfrac{(x^2+\sin x)\cos x-\sin x(2x+\cos x)}{(x^2+\sin x)^2}=\dfrac{x^2\cos x-2x\sin x}{(x^2+\sin x)^2}$

7. $f'(x)=\sec x\tan x-\sqrt{2}\sec^2 x$

8. $f'(x)=(x^2+1)\sec x\tan x+(\sec x)(2x)=(x^2+1)\sec x\tan x+2x\sec x$

9. $f'(x) = -4\csc x \cot x + \csc^2 x$

10. $f'(x) = -\sin x - \csc x + x\csc x \cot x$

11. $f'(x) = \sec x(\sec^2 x) + (\tan x)(\sec x \tan x) = \sec^3 x + \sec x \tan^2 x$

12. $f'(x) = (\csc x)(-\csc^2 x) + (\cot x)(-\csc x \cot x) = -\csc^3 x - \csc x \cot^2 x$

13. $f'(x) = \dfrac{(1+\csc x)(-\csc^2 x) - \cot x(0 - \csc x \cot x)}{(1+\csc x)^2} = \dfrac{\csc x(-\csc x - \csc^2 x + \cot^2 x)}{(1+\csc x)^2}$ but

$1 + \cot^2 x = \csc^2 x$ (identity) thus $\cot^2 x - \csc^2 x = -1$ so

$f'(x) = \dfrac{\csc x(-\csc x - 1)}{(1+\csc x)^2} = -\dfrac{\csc x}{1+\csc x}$

14. $f'(x) = \dfrac{(1+\tan x)(\sec x \tan x) - (\sec x)(\sec^2 x)}{(1+\tan x)^2} = \dfrac{\sec x \tan x + \sec x \tan^2 x - \sec^3 x}{(1+\tan x)^2}$

$= \dfrac{\sec x(\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2} = \dfrac{\sec x(\tan x - 1)}{(1+\tan x)^2}$

15. $f(x) = \sin^2 x + \cos^2 x = 1$ (identity) so $f'(x) = 0$

16. $f'(x) = 2\sec x \tan x \sec x - 2\tan x \sec^2 x = \dfrac{2\sin x}{\cos^3 x} - 2\dfrac{\sin x}{\cos^3 x} = 0$

OR $f(x) = \sec^2 x - \tan^2 x = 1$ (identity), $f'(x) = 0$

17. $f(x) = \dfrac{\tan x}{1 + x\tan x}$ (because $\sin x \sec x = (\sin x)(1/\cos x) = \tan x$),

$f'(x) = \dfrac{(1 + x\tan x)(\sec^2 x) - \tan x[x(\sec^2 x) + (\tan x)(1)]}{(1 + x\tan x)^2}$

$= \dfrac{\sec^2 x - \tan^2 x}{(1 + x\tan x)^2} = \dfrac{1}{(1 + x\tan x)^2}$ (because $\sec^2 x - \tan^2 x = 1$)

18. $f(x) = \dfrac{(x^2+1)\cot x}{3 - \cot x}$ (because $\cos x \csc x = (\cos x)(1/\sin x) = \cot x$),

$f'(x) = \dfrac{(3 - \cot x)[2x\cot x - (x^2+1)\csc^2 x] - (x^2+1)\cot x \csc^2 x}{(3 - \cot x)^2}$

$= \dfrac{6x\cot x - 2x\cot^2 x - 3(x^2+1)\csc^2 x}{(3 - \cot x)^2}$

19. $dy/dx = -x\sin x + \cos x$, $d^2y/dx^2 = -x\cos x - \sin x - \sin x = -x\cos x - 2\sin x$

20. $dy/dx = -\csc x \cot x$, $d^2y/dx^2 = -[(\csc x)(-\csc^2 x) + (\cot x)(-\csc x \cot x)] = \csc^3 x + \csc x \cot^2 x$

21. $dy/dx = x(\cos x) + (\sin x)(1) - 3(-\sin x) = x\cos x + 4\sin x$,
$d^2y/dx^2 = x(-\sin x) + (\cos x)(1) + 4\cos x = -x\sin x + 5\cos x$

22. $dy/dx = x^2(-\sin x) + (\cos x)(2x) + 4\cos x = -x^2 \sin x + 2x\cos x + 4\cos x$,
$d^2y/dx^2 = -[x^2(\cos x) + (\sin x)(2x)] + 2[x(-\sin x) + \cos x] - 4\sin x = (2 - x^2)\cos x - 4(x+1)\sin x$

23. $dy/dx = (\sin x)(-\sin x) + (\cos x)(\cos x) = \cos^2 x - \sin^2 x$,
$d^2y/dx^2 = (\cos x)(-\sin x) + (\cos x)(-\sin x) - [(\sin x)(\cos x) + (\sin x)(\cos x)] = -4\sin x \cos x$

24. $dy/dx = \sec^2 x$; $d^2y/dx^2 = 2\sec^2 x \tan x$

25. Let $f(x) = \tan x$, then $f'(x) = \sec^2 x$.
   (a) $f(0) = 0$ and $f'(0) = 1$ so $y - 0 = (1)(x - 0)$, $y = x$.
   (b) $f\left(\frac{\pi}{4}\right) = 1$ and $f'\left(\frac{\pi}{4}\right) = 2$ so $y - 1 = 2\left(x - \frac{\pi}{4}\right)$, $y = 2x - \frac{\pi}{2} + 1$.
   (c) $f\left(-\frac{\pi}{4}\right) = -1$ and $f'\left(-\frac{\pi}{4}\right) = 2$ so $y + 1 = 2\left(x + \frac{\pi}{4}\right)$, $y = 2x + \frac{\pi}{2} - 1$.

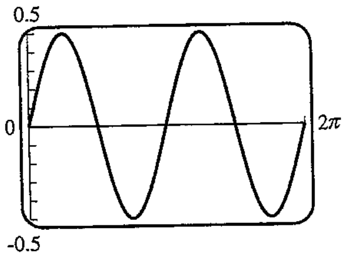
26. Let $f(x) = \sin x$, then $f'(x) = \cos x$.
   (a) $f(0) = 0$ and $f'(0) = 1$ so $y - 0 = (1)(x - 0)$, $y = x$
   (b) $f(\pi) = 0$ and $f'(\pi) = -1$ so $y - 0 = (-1)(x - \pi)$, $y = -x + \pi$
   (c) $f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $f'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ so $y - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)$, $y = \frac{1}{\sqrt{2}}x - \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}}$

27. (a) If $y = x\sin x$ then $y' = \sin x + x\cos x$ and $y'' = 2\cos x - x\sin x$ so $y'' + y = 2\cos x$.
   (b) If $y = x\sin x$ then $y' = \sin x + x\cos x$ and $y'' = 2\cos x - x\sin x$ so $y'' + y = 2\cos x$; differentiate twice more to get $y^{(4)} + y'' = -2\cos x$.

28. (a) If $y = \cos x$ then $y' = -\sin x$ and $y'' = -\cos x$ so $y'' + y = (-\cos x) + (\cos x) = 0$; if $y = \sin x$ then $y' = \cos x$ and $y'' = -\sin x$ so $y'' + y = (-\sin x) + (\sin x) = 0$.
   (b) $y' = A\cos x - B\sin x$, $y'' = -A\sin x - B\cos x$ so $y'' + y = (-A\sin x - B\cos x) + (A\sin x + B\cos x) = 0$.

29. (a) $f'(x) = \cos x = 0$ at $x = \pm\pi/2, \pm 3\pi/2$.
   (b) $f'(x) = 1 - \sin x = 0$ at $x = -3\pi/2, \pi/2$.
   (c) $f'(x) = \sec^2 x \geq 1$ always, so no horizontal tangent line.
   (d) $f'(x) = \sec x \tan x = 0$ when $\sin x = 0$, $x = \pm 2\pi, \pm\pi, 0$

30. (a)



   (b) $y = \sin x \cos x = (1/2)\sin 2x$ and $y' = \cos 2x$. So $y' = 0$ when $2x = (2n+1)\pi/2$ for $n = 0, 1, 2, 3$ or $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

31. $x = 10\sin\theta$, $dx/d\theta = 10\cos\theta$; if $\theta = 60°$, then $dx/d\theta = 10(1/2) = 5$ ft/rad $= \pi/36$ ft/deg ≈ 0.087 ft/deg

32. $s = 3800\csc\theta, ds/d\theta = -3800\csc\theta\cot\theta$; if $\theta = 30°$, then $ds/d\theta = -3800(2)(\sqrt{3}) = -7600\sqrt{3}$ ft/rad $= -380\sqrt{3}\pi/9$ ft/deg ≈ -230 ft/deg

33. $D = 50\tan\theta, dD/d\theta = 50\sec^2\theta$; if $\theta = 45°$, then $dD/d\theta = 50(\sqrt{2})^2 = 100$ m/rad $= 5\pi/9$ m/deg ≈ 1.75 m/deg

34. (a) From the right triangle shown, $\sin\theta = r/(r+h)$ so $r + h = r\csc\theta$, $h = r(\csc\theta - 1)$.
   (b) $dh/d\theta = -r\csc\theta\cot\theta$; if $\theta = 30°$, then $dh/d\theta = -6378(2)(\sqrt{3}) \approx -22,094$ km/rad ≈ -386 km/deg

35. (a) $\dfrac{d^4}{dx^4}\sin x = \sin x$, so $\dfrac{d^{4k}}{dx^{4k}}\sin x = \sin x$; $\dfrac{d^{87}}{dx^{87}}\sin x = \dfrac{d^3}{dx^3}\dfrac{d^{4\cdot 21}}{dx^{4\cdot 21}}\sin x = \dfrac{d^3}{dx^3}\sin x = -\cos x$

(b) $\dfrac{d^{100}}{dx^{100}}\cos x = \dfrac{d^{4k}}{dx^{4k}}\cos x = \cos x$

36. $\dfrac{d}{dx}[x\sin x] = x\cos x + \sin x$ $\qquad \dfrac{d^2}{dx^2}[x\sin x] = -x\sin x + 2\cos x$

$\dfrac{d^3}{dx^3}[x\sin x] = -x\cos x - 3\sin x$ $\qquad \dfrac{d^4}{dx^4}[x\sin x] = x\sin x - 4\cos x$

By mathematical induction one can show

$\dfrac{d^{4k}}{dx^{4k}}[x\sin x] = x\sin x - (4k)\cos x$; $\qquad \dfrac{d^{4k+1}}{dx^{4k+1}}[x\sin x] = x\cos x + (4k+1)\sin x$;

$\dfrac{d^{4k+2}}{dx^{4k+2}}[x\sin x] = -x\sin x + (4k+2)\cos x$; $\qquad \dfrac{d^{4k+3}}{dx^{4k+3}}[x\sin x] = -x\cos x - (4k+3)\sin x$;

Since $17 = 4\cdot 4 + 1$, $\dfrac{d^{17}}{dx^{17}}[x\sin x] = x\cos x + 17\sin x$

37. $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$, and $f^{(4)}(x) = \cos x$ with higher order derivatives repeating this pattern, so $f^{(n)}(x) = \sin x$ for $n = 3, 7, 11, \ldots$

38. $f(x) = \sin x$, $f'(x) = \cos x$, $f''(x) = -\sin x$, $f^{(4)}(x) = -\cos x$, $f^{(5)}(x) = \sin x$, and the right-hand sides continue with a period of 4, so that $f^{(n)}(x) = \sin x$ when $n = 4k$ for some $k > 0$.

39. (a) all x $\qquad$ (b) all x
(c) $x \neq \pi/2 + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ $\qquad$ (d) $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$
(e) $x \neq \pi/2 + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ $\qquad$ (f) $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$
(g) $x \neq (2n+1)\pi$, $n = 0, \pm 1, \pm 2, \ldots$ $\qquad$ (h) $x \neq n\pi/2$, $n = 0, \pm 1, \pm 2, \ldots$
(i) all x

40. (a) $$\frac{d}{dx}[\cos x] = \lim_{h\to 0}\frac{\cos(x+h) - \cos x}{h} = \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$
$$= \lim_{h\to 0}\left[\cos x\left(\frac{\cos h - 1}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)\right] = (\cos x)(0) - (\sin x)(1) = -\sin x$$

(b) $$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$$
$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

(c) $$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{0\cdot\cos x - (1)(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \sec x\tan x$$

(d) $$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{(\sin x)(0) - (1)(\cos x)}{\sin^2 x} = -\frac{\cos x}{\sin^2 x} = -\csc x\cot x$$