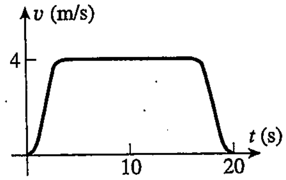
CHAPTER 3
The Derivative

EXERCISE SET 3.1

1. (a) $m_{\tan} = (50 - 10)/(15 - 5)$
$= 40/10$
$= 4$ m/s

(b)


2. (a) $m_{\tan} \approx (90 - 0)/(10 - 2)$
$= 90/8$
$= 11.25$ m/s

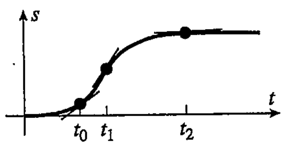
(b) $m_{\tan} \approx (140 - 0)/(10 - 4)$
$= 140/6$
≈ 23.33 m/s

3. (a) $m_{\tan} = (600 - 0)/(20 - 2.2)$
$= 600/17.8$
≈ 33.71 m/s

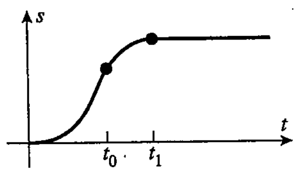
(b) $m_{\tan} \approx (820 - 600)/(20 - 16)$
$= 220/4$
$= 55$ m/s
The speed is increasing with time.

4. (a) $(10 - 10)/(3 - 0) = 0$ cm/s
(b) $t = 0$, $t = 2$, and $t = 4.2$ (horizontal tangent line)
(c) maximum: $t = 1$ (slope > 0) minimum: $t = 3$ (slope < 0)
(d) $(3 - 18)/(4 - 2) = -7.5$ cm/s (slope of estimated tangent line to curve at $t = 3$)

5. From the figure:



(a) The particle is moving faster at time t_0 because the slope of the tangent to the curve at t_0 is greater than that at t_2.
(b) The initial velocity is 0 because the slope of a horizontal line is 0.
(c) The particle is speeding up because the slope increases as t increases from t_0 to t_1.
(d) The particle is slowing down because the slope decreases as t increases from t_1 to t_2.

6.


7. It is a straight line with slope equal to the velocity.

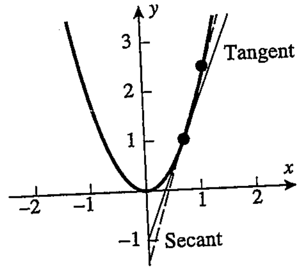
8. (a) decreasing (slope of tangent line decreases with increasing time)
(b) increasing (slope of tangent line increases with increasing time)
(c) increasing (slope of tangent line increases with increasing time)
(d) decreasing (slope of tangent line decreases with increasing time)

9. (a) $m_{\text{sec}} = \dfrac{f(1) - f(0)}{1 - 0} = \dfrac{2}{1} = 2$

(b) $m_{\tan} = \lim\limits_{x_1 \to 0} \dfrac{f(x_1) - f(0)}{x_1 - 0} = \lim\limits_{x_1 \to 0} \dfrac{2x_1^2 - 0}{x_1 - 0} = \lim\limits_{x_1 \to 0} 2x_1 = 0$

(c)
$$m_{\tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{2x_1^2 - 2x_0^2}{x_1 - x_0} = \lim_{x_1 \to x_0} (2x_1 + 2x_0) = 4x_0$$

(d)

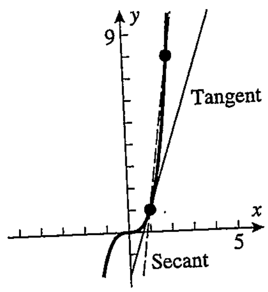


10. (a) $m_{\text{sec}} = \dfrac{f(2) - f(1)}{2 - 1} = \dfrac{2^3 - 1^3}{1} = 7$

(b)
$$m_{\tan} = \lim_{x_1 \to 1} \frac{f(x_1) - f(1)}{x_1 - 1} = \lim_{x_1 \to 1} \frac{x_1^3 - 1}{x_1 - 1} = \lim_{x_1 \to 1} \frac{(x_1 - 1)(x_1^2 + x_1 + 1)}{x_1 - 1} = \lim_{x_1 \to 1} (x_1^2 + x_1 + 1) = 3$$

(c)
$$m_{\tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{x_1^3 - x_0^3}{x_1 - x_0} = \lim_{x_1 \to x_0} (x_1^2 + x_1x_0 + x_0^2) = 3x_0^2$$

(d)

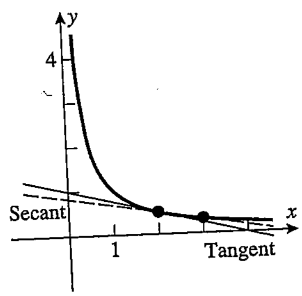


11. (a) $m_{\text{sec}} = \dfrac{f(3) - f(2)}{3 - 2} = \dfrac{1/3 - 1/2}{1} = -\dfrac{1}{6}$

(b)
$$m_{\tan} = \lim_{x_1 \to 2} \frac{f(x_1) - f(2)}{x_1 - 2} = \lim_{x_1 \to 2} \frac{1/x_1 - 1/2}{x_1 - 2} = \lim_{x_1 \to 2} \frac{2 - x_1}{2x_1(x_1 - 2)} = \lim_{x_1 \to 2} \frac{-1}{2x_1} = -\frac{1}{4}$$

(c)
$$m_{\tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{1/x_1 - 1/x_0}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{x_0 - x_1}{x_0x_1(x_1 - x_0)} = \lim_{x_1 \to x_0} \frac{-1}{x_0x_1} = -\frac{1}{x_0^2}$$

(d)



12. (a) $m_{\text{sec}} = \dfrac{f(2)-f(1)}{2-1} = \dfrac{1/4-1}{1} = -\dfrac{3}{4}$

(b) $m_{\tan} = \lim\limits_{x_1\to 1} \dfrac{f(x_1)-f(1)}{x_1-1} = \lim\limits_{x_1\to 1} \dfrac{1/x_1^2-1}{x_1-1}$

$= \lim\limits_{x_1\to 1} \dfrac{1-x_1^2}{x_1^2(x_1-1)} = \lim\limits_{x_1\to 1} \dfrac{-(x_1+1)}{x_1^2} = -2$

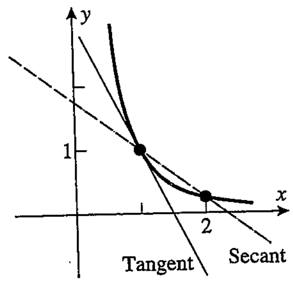
(c) $m_{\tan} = \lim\limits_{x_1\to x_0} \dfrac{f(x_1)-f(x_0)}{x_1-x_0}$

$= \lim\limits_{x_1\to x_0} \dfrac{1/x_1^2-1/x_0^2}{x_1-x_0}$

$= \lim\limits_{x_1\to x_0} \dfrac{x_0^2-x_1^2}{x_0^2x_1^2(x_1-x_0)}$

$= \lim\limits_{x_1\to x_0} \dfrac{-(x_1+x_0)}{x_0^2x_1^2} = -\dfrac{2}{x_0^3}$

(d)



13. (a) $m_{\tan} = \lim\limits_{x_1\to x_0} \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \lim\limits_{x_1\to x_0} \dfrac{(x_1^2-1)-(x_0^2-1)}{x_1-x_0}$

$= \lim\limits_{x_1\to x_0} \dfrac{(x_1^2-x_0^2)}{x_1-x_0} = \lim\limits_{x_1\to x_0} (x_1+x_0) = 2x_0$

(b) $m_{\tan} = 2(-1) = -2$

14. (a) $m_{\tan} = \lim\limits_{x_1\to x_0} \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \lim\limits_{x_1\to x_0} \dfrac{(x_1^2+3x_1+2)-(x_0^2+3x_0+2)}{x_1-x_0}$

$= \lim\limits_{x_1\to x_0} \dfrac{(x_1^2-x_0^2)+3(x_1-x_0)}{x_1-x_0} = \lim\limits_{x_1\to x_0} (x_1+x_0+3) = 2x_0+3$

(b) $m_{\tan} = 2(2)+3 = 7$

15. (a) $m_{\tan} = \lim\limits_{x_1\to x_0} \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \lim\limits_{x_1\to x_0} \dfrac{\sqrt{x_1}-\sqrt{x_0}}{x_1-x_0}$

$= \lim\limits_{x_1\to x_0} \dfrac{1}{\sqrt{x_1}+\sqrt{x_0}} = \dfrac{1}{2\sqrt{x_0}}$

(b) $m_{\tan} = \dfrac{1}{2\sqrt{1}} = \dfrac{1}{2}$

16. (a) $m_{\tan} = \lim\limits_{x_1\to x_0} \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \lim\limits_{x_1\to x_0} \dfrac{1/\sqrt{x_1}-1/\sqrt{x_0}}{x_1-x_0}$

$= \lim\limits_{x_1\to x_0} \dfrac{\sqrt{x_0}-\sqrt{x_1}}{\sqrt{x_0}\,\sqrt{x_1}\,(x_1-x_0)} = \lim\limits_{x_1\to x_0} \dfrac{-1}{\sqrt{x_0}\,\sqrt{x_1}\,(\sqrt{x_1}+\sqrt{x_0})} = -\dfrac{1}{2x_0^{3/2}}$

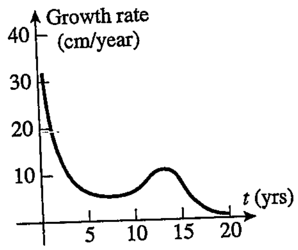
(b) $m_{\tan} = -\dfrac{1}{2(4)^{3/2}} = -\dfrac{1}{16}$

17. (a) 72°F at about 4:30 P.M. (b) about $(67-43)/6 = 4$°F/h

(c) decreasing most rapidly at about 9 P.M.; rate of change of temperature is about −7°F/h (slope of estimated tangent line to curve at 9 P.M.)

18. For $V = 10$ the slope of the tangent line is about -0.25 atm/L, for $V = 25$ the slope is about -0.04 atm/L.

19. (a) during the first year after birth
(b) about 6 cm/year (slope of estimated tangent line at age 5)
(c) the growth rate is greatest at about age 14; about 10 cm/year
(d)



20. (a) The rock will hit the ground when $16t^2 = 576$, $t^2 = 36$, $t = 6$ s (only $t \geq 0$ is meaningful)

(b) $v_{\text{ave}} = \dfrac{16(6)^2 - 16(0)^2}{6-0} = 96$ ft/s

(c) $v_{\text{ave}} = \dfrac{16(3)^2 - 16(0)^2}{3-0} = 48$ ft/s

(d) $v_{\text{inst}} = \lim\limits_{t_1 \to 6} \dfrac{16t_1^2 - 16(6)^2}{t_1 - 6} = \lim\limits_{t_1 \to 6} \dfrac{16(t_1^2 - 36)}{t_1 - 6}$

$= \lim\limits_{t_1 \to 6} 16(t_1 + 6) = 192$ ft/s

21. (a) $(40)^3/\sqrt{10} = 20{,}238.6$ ft (b) $v_{\text{ave}} = 20{,}238.6/40 = 505.96$ ft/s

(c) Solve $s = t^3/\sqrt{10} = 135$, $t \approx 7.53$ so $v_{\text{ave}} = 135/7.53 = 17.93$ ft/s.

(d) $v_{\text{inst}} = \lim\limits_{t_1 \to 40} \dfrac{t_1^3/\sqrt{10} - (40)^3/\sqrt{10}}{t_1 - 40} = \lim\limits_{t_1 \to 40} \dfrac{(t_1^3 - 40^3)}{(t_1 - 40)\sqrt{10}}$

$= \lim\limits_{t_1 \to 40} \dfrac{1}{\sqrt{10}}(t_1^2 + 40t_1 + 1600) = 1517.89$ ft/s

22. (a) $v_{\text{ave}} = \dfrac{4.5(12)^2 - 4.5(0)^2}{12-0} = 54$ ft/s

(b) $v_{\text{inst}} = \lim\limits_{t_1 \to 6} \dfrac{4.5t_1^2 - 4.5(6)^2}{t_1 - 6} = \lim\limits_{t_1 \to 6} \dfrac{4.5(t_1^2 - 36)}{t_1 - 6}$

$= \lim\limits_{t_1 \to 6} \dfrac{4.5(t_1 + 6)(t_1 - 6)}{t_1 - 6} = \lim\limits_{t_1 \to 6} 4.5(t_1 + 6) = 54$ ft/s

23. (a) $v_{\text{ave}} = \dfrac{6(4)^4 - 6(2)^4}{4-2} = 720$ ft/min

(b) $v_{\text{inst}} = \lim\limits_{t_1 \to 2} \dfrac{6t_1^4 - 6(2)^4}{t_1 - 2} = \lim\limits_{t_1 \to 2} \dfrac{6(t_1^4 - 16)}{t_1 - 2}$

$= \lim\limits_{t_1 \to 2} \dfrac{6(t_1^2 + 4)(t_1^2 - 4)}{t_1 - 2} = \lim\limits_{t_1 \to 2} 6(t_1^2 + 4)(t_1 + 2) = 192$ ft/min