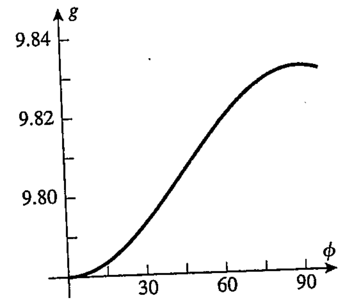
77. **(a)** Gravity is stronger at the poles than at the equator.



(b) Let $g(\phi)$ be the given function. Then $g(38) < 9.8$ and $g(39) > 9.8$, so by the Intermediate Value Theorem there is a value c between 38 and 39 for which $g(c) = 9.8$ exactly.

78. **(a)** does not exist **(b)** the limit is zero

(c) For part (a) consider the fact that given any $\delta > 0$ there are infinitely many rational numbers x satisfying $|x| < \delta$ and there are infinitely many irrational numbers satisfying the same condition. Thus if the limit were to exist, it could not be zero because of the rational numbers, and it could not be 1 because of the irrational numbers, and it could not be anything else because of *all* the numbers. Hence the limit cannot exist. For part (b) use the Squeezing Theorem with $+x$ and $-x$ as the 'squeezers'.

REVIEW EXERCISES, CHAPTER 2

1. **(a)** 1 **(b)** no limit **(c)** no limit

(d) 1 **(e)** 3 **(f)** 0

(g) 0 **(h)** 2 **(i)** 1/2

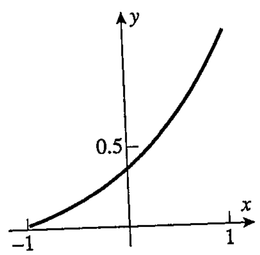
2. **(a)** $0.222\ldots, 0.24390, 0.24938, 0.24994, 0.24999, 0.25000$; for $x \neq 2$, $f(x) = \dfrac{1}{x+2}$, so the limit is 1/4.

(b) $1.15782, 4.22793, 4.00213, 4.00002, 4.00000, 4.00000$; to prove, use $\dfrac{\tan 4x}{x} = \dfrac{\sin 4x}{x \cos 4x} = \dfrac{4}{\cos 4x}\dfrac{\sin 4x}{4x}$, the limit is 4.

3. **(a)**

x	1	0.1	0.01	0.001	0.0001	0.00001	0.000001
$f(x)$	1.000	0.443	0.409	0.406	0.406	0.405	0.405

(b)



4.

x	3.1	3.01	3.001	3.0001	3.00001	3.000001
$f(x)$	5.74	5.56	5.547	5.545	5.5452	5.54518

A CAS yields 5.545177445

5. 1 1.

6. For $x \neq 1$, $\dfrac{x^3 - x^2}{x - 1} = x^2$, so $\lim\limits_{x \to -1} \dfrac{x^3 - x^2}{x - 1} = 1$

7. If $x \neq -3$ then $\dfrac{3x+9}{x^2+4x+3} = \dfrac{3}{x+1}$ with limit $-\dfrac{3}{2}$

8. $-\infty$

9. $\dfrac{2^5}{3} = \dfrac{32}{3}$

10. $\dfrac{\sqrt{x^2+4}-2}{x^2} \dfrac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2} = \dfrac{x^2}{x^2(\sqrt{x^2+4}+2)} = \dfrac{1}{\sqrt{x^2+4}+2}$, so

$$\lim_{x \to 0} \frac{\sqrt{x^2+4}-2}{x^2} = \lim_{x \to 0} \frac{1}{\sqrt{x^2+4}+2} = \frac{1}{4}$$

11. **(a)** $y = 0$ **(b)** none **(c)** $y = 2$

12. **(a)** $\sqrt{5}$, no limit, $\sqrt{10}$, $\sqrt{10}$, no limit, $+\infty$, no limit
(b) $-1, +1, -1, -1$, no limit, $-1, +1$

13. 1

14. 2

15. $3 - k$

16. $\lim\limits_{\theta \to 0} \tan\left(\dfrac{1-\cos\theta}{\theta}\right) = \tan\left(\lim\limits_{\theta \to 0} \dfrac{1-\cos\theta}{\theta}\right) = \tan\left(\lim\limits_{\theta \to 0} \dfrac{1-\cos^2\theta}{\theta(1+\cos\theta)}\right) = \tan 0 = 0$

17. $+\infty$

18. $\ln(2\sin\theta\cos\theta) - \ln\tan\theta = \ln 2 + 2\ln\cos\theta$ so the limit is $\ln 2$.

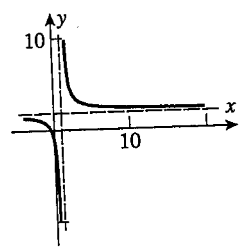
19. $\left(1+\dfrac{3}{x}\right)^{-x} = \left[\left(1+\dfrac{3}{x}\right)^{x/3}\right]^{(-3)}$ so the limit is e^{-3}

20. $\left(1+\dfrac{a}{x}\right)^{bx} = \left[\left(1+\dfrac{a}{x}\right)^{x/a}\right]^{(ab)}$ so the limit is e^{ab}

21. \$2,001.60, \$2,009.66, \$2,013.62, \$2013.75

23. **(a)** $f(x) = 2x/(x-1)$

(b)



24. Given any window of height 2ϵ centered at the point $x = a, y = L$ there exists a width 2δ such that the window of width 2δ and height 2ϵ contains all points of the graph of the function for x in that interval.

25. **(a)** $\lim\limits_{x \to 2} f(x) = 5$ **(b)** 0.0045

26. $\delta \approx 0.07747$ (use a graphing utility)

27. (a) $|4x-7-1|<0.01, 4|x-2|<0.01, |x-2|<0.0025$, let $\delta=0.0025$

(b) $\left|\dfrac{4x^2-9}{2x-3}-6\right|<0.05, |2x+3-6|<0.05, |x-1.5|<0.025$, take $\delta=0.025$

(c) $|x^2-16|<0.001$; if $\delta<1$ then $|x+4|<9$ if $|x-4|<1$; then $|x^2-16|=|x-4||x+4|\le 9|x-4|<0.001$ provided $|x-4|<0.001/9$, take $\delta=0.0001$, then $|x^2-16|<9|x-4|<9(0.0001)=0.0009<0.001$

28. (a) Given $\epsilon>0$ then $|4x-7-1|<\epsilon$ provided $|x-2|<\epsilon/4$, take $\delta=\epsilon/4$

(b) Given $\epsilon>0$ the inequality $\left|\dfrac{4x^2-9}{2x-3}-6\right|<\epsilon$ holds if $|2x+3-6|<\epsilon, |x-1.5|<\epsilon/2$, take $\delta=\epsilon/2$

29. Let $\epsilon=f(x_0)/2>0$; then there corresponds $\delta>0$ such that if $|x-x_0|<\delta$ then $|f(x)-f(x_0)|<\epsilon$, $-\epsilon<f(x)-f(x_0)<\epsilon$, $f(x)>f(x_0)-\epsilon=f(x_0)/2>0$ for $x_0-\delta<x<x_0+\delta$.

30. (a)

x	1.1	1.01	1.001	1.0001	1.00001	1.000001
$f(x)$	0.49	0.54	0.540	0.5403	0.54030	0.54030

(b) $\cos 1$

31. (a) f is not defined at $x=\pm1$, continuous elsewhere
(b) none
(c) f is not defined at $x=0,-3$

32. (a) continuous everywhere except $x=\pm3$
(b) defined and continuous for $x\le-1$, $x\ge1$
(c) continuous for $x>0$

33. For $x<2$ f is a polynomial and is continuous; for $x>2$ f is a polynomial and is continuous. At $x=2$, $f(2)=-13\ne13=\lim\limits_{x\to2^+}f(x)$ so f is not continuous there.

35. $f(x)=-1$ for $a\le x<\dfrac{a+b}{2}$ and $f(x)=1$ for $\dfrac{a+b}{2}\le x\le b$

36. If, on the contrary, $f(x_0)<0$ for some x_0 in $[0,1]$, then by the Intermediate Value Theorem we would have a solution of $f(x)=0$ in $[0,x_0]$, contrary to the hypothesis.

37. $f(-6)=185$, $f(0)=-1$, $f(2)=65$; apply Theorem 2.4.8 twice, once on $[-6,0]$ and once on $[0,2]$