Applications of Derivatives: Finding Maxima and Minima

In Unit 2, we have learned how to differentiate any function. In Unit 3, we will be thinking about how to apply the derivative to a variety of situations.

GroupWork

We are already aware that the derivative gives us qualitative information about the shape of the function. You may recall some of the following concepts. Begin by filling in the necessary information below:

function	**derivative**	**illustration**
increasing		
	negative	
level (flat)		
	undefined	
steep (rising or falling)		
	small (positive or negative)	
straight (horizontal)		
straight (slanted)		

Critical Point A point $(c, f(c))$ is called a *critical point* of a function $f(x)$ if $f'(c) = 0$ or $f'(c)$ does not exist.

Extremum The maximum or minimum value a function $f(x)$ outputs on a particular domain is called an **extremum** or extreme value. The plural is extrema.

The <u>First Derivative Test</u> for finding Local Extrema Let $(c, f(c))$ be a critical point in the domain of $f(x)$:

> If $f'(x)$ is negative to the left of c and positive to the right of c, then $f(x)$ has a local minimum at c.
>
> If $f'(x)$ is positive to the left of c and negative to the right of c, then $f(x)$ has a local maximum at c.

Global Minimum and Global Maximum The idea of finding global extrema is very important in word problems.

$f(x)$ has a *global minimum* at p if all values of $f(x)$ are greater than or equal to $f(p)$.

$f(x)$ has a *global maximum* at p if all values of $f(x)$ are less than or equal to $f(p)$.

How to find Local Extremes

1. Determine the domain of the function and identify the end points (if any).
2. Find $f^{'}(x)$.
3. Find all roots of $f^{'}(x) = 0$ in the domain, and find where $f^{'}(x)$ does not exist.
4. Use the First Derivative Test to locate any local extrema.

How to find Global Extremes

1. Determine Local extremes $(c, f(c))$, as above.
2. If the $Domain = [a, b]$, then check the end point values $f(a)$ and $f(b)$ and determine which value among $\{f(c), f(a), f(b)\}$ is the largest - this is the global maximum; the smallest is the global minimum.
3. If you do not have a closed interval for the domain, then the largest local maximum is the global maximum, and the smallest local minimum is the global minimum.

Example 1 Find the intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing.

Interval			
Test value			
Sign of $f'(x)$			
Conclusion			

Example 2. Find the relative extrema of the function $f(x) = x^4 - 6x^2 + 7$ and draw a sketch of its graph