Solutions of Initial Value Problems

Checking Solutions of Rate Equations

Is $y(t) = e^{2t}$ a solution of the rate equation $y'(t) = y(t)$?

$$y'(t) = \frac{d}{dt}e^{2t} = e^{2t} \cdot \frac{d}{dt}2t = 2e^{2t} = 2\,y(t) \neq y(t).$$

Thus the answer to this question is "NO." Can you think of a function $y(t)$ which DOES satisfy this rate equation? If so, check your answer below.

Can you think of ANOTHER function which satisfies $y'(t) = y(t)$? (*Hint:* Think about the Constant Multiple Rule.) Check your guess below.

Checking Initial Conditions

Does $y(t) = 2\,e^t$ satisfy the initial condition $y(0) = 2$?

$$y(0) = 2\,e^0 = 2 \cdot 1 = 2.$$

Thus, the answer to this question is "YES."

Checking Solutions of Initial Value Problems

Based on your work above, is $y(t) = 2\,e^t$ a solution of the following initial value problem?

$$y'(t) = y(t), \quad y(0) = 2$$

Explain.

Consider the following initial value problem:

$$C'(t) = -t \cdot C(t), \qquad C(0) = 1$$

In Math 120 (Integral Calculus) you will learn how to solve this initial value problem and derive the solution

$$C(t) = e^{-\frac{t^2}{2}}$$

However, we can use our ability to differentiate functions to check whether a given proposed solution to an initial value problem is actually a true solution.

Given the proposed solution $C(t) = e^{-\frac{t^2}{2}}$:

1. Evaluate $C(0)$

2. Find $C'(t)$

3. Does the proposed $C(t)$ satisfy the given initial value problem?

Example

4. The function $y(x) = 2x\,(\ln(x) - 1)$ is proposed as a solution to the following IVP:

$$y'(x) = \ln(x^2), \qquad y(1) = 0.$$

Does $y(x)$ satisfy the initial value problem? If not, can you modify it so that your modified version does satisfy the entire initial value problem?