Reviewing the First Unit

The following statements are either true or false. Decided, then explain your answer, illustrating with examples if possible.

1. The rule for every function must be expressed as a formula.

2. Every direct proportionality is a linear function but not every linear function is a direct proportionality.

Answer the following questions:

3. What is the relationship between a rate equation and its slope field? If a rate y'(x) depends explicitly (in the rate equation) only on y what special feature does its slope field have? If a rate y'(x) depends explicitly (in the rate equation) only on x what special feature does its slope field have?

4. Given a slope field for a rate equation, how can you use it to trace out the graphs of approximate solutions? How is Euler's Method using the slope field to create a piecewise linear approximation?

5. Draw a diagram illustrating the way Euler's Method estimates the point $(t + \Delta t, y(t + \Delta t))$ given the point (t, y(t)), the stepsize Δt and the slope y'(t).

6. Why does using a smaller stepsize with Euler's Method give you a more accurate approximation? Also give two reasons why successive approximation better than just running Euler's Method once with a small stepsize.

Solve the following problem:

7. Use Euler's Method with stepsize $\Delta t = 0.5$ to construct a piecewise linear approximation to the solution of the following initial value problem on the interval $0 \le t \le 1$. Then find the formula for the last line segment in this approximation and use this formula to evaluate the approximation at t = 0.8.

$$y'(t) = t + y,$$
 $y(0) = 0.$