Analyzing the S-I-R Model

1. What does it mean to have an *epidemic*?

So, to in mathematical notation, to say we have an epidemic we can write

$$I'$$

The general form of the S-I-R model can be written:

$$\begin{aligned} S'(t) &= -aSI \\ I'(t) &= aSI - bI \\ R'(t) &= bI \end{aligned}$$

The constants a and b are parameters of the model, where a related to the ease at which the disease is transmitted through the population and b is related to the length of the recovery period. In fact b is simply the *reciprocal* of the duration of the disease (in other words $1/b$ is how long it takes to recover from the illness.)
On the other hand, a, known as the *transmission coefficient* is a product of the infectiousness of the disease and the level of social interaction or "mixing" of the population.

2. The specific example we approximated solutions for was
$S'(t) = -.00001SI$
$I'(t) = .00001SI - I/14$
$R'(t) = I/14$

Given the present values:
$S = 45400$, $I = 2100$, $R = 2500$,
how can we tell whether or not we are currently experiencing an epidemic?

3. What if S was 5000, with everything else unchanged; would there be an epidemic?

4. What if S was 10000, with everything else unchanged; would there be an epidemic?

5. What is the HIGHEST possible value for S that will NOT turn into an epidemic?

This value is called the *threshold value* for an epidemic. Write down the formula for the threshold value for the general S-I-R model.

GROUPWORK

6. Q: How would you modify the model in (2) if the recovery period for the disease was one week?

7. Q: Suppose a disease in the town of Hoosville is modelled by the equations:
$S' = -.00002SI$, $I' = .00002SI - 0.1I$, and $R' = 0.1I$.
How long is the recovery period for this disease?

8. Q: How would the above model be modified if the weather in the town became lousy and thus the number of people "out and about" in Hoosville every day was cut in half? Is the effect on the model the same or different from a quarantine?

9. Q: Suppose 10 percent of the population has been vaccinated and can not catch the disease, how would you modify your model?

10. Q: Suppose after a certain amount of time, say M days, people who have recovered from the disease *lose their immunity* and become susceptible again. How would the model have to be modified? (For more on the idea of immunity loss, see problems 23-26 in CiC.)