S-I-R Model of Disease

Suppose we want to model the spread of an infectious disease (like measles).

Simplifying assumptions:
– Nobody dies from it!
– Recovery always takes 14 days.
– You're contagious during those 14 days.
– You cannot get it twice.

Notation:
I = number of infected people.
R = number of recovered people (i.e., already had it).
S = number of susceptible people (i.e., haven't had it yet).

Rates of change: $I'(t)$, $R'(t)$, $S'(t)$.
Units: ________________ per day.

Q: If I people are currently infected, how many of them do you expect will recover today? ______.

So,

$$R'(t) =$$

True of false?
$I'(t)$ = number of people who get infected per day.
$S'(t)$ = -(number of people who get infected per day).

To write an equation for $S'(t)$, first note that on any given day, the number of people who get infected depends on the number of susceptible people who come into contact with infected people:
–If everything else was the same except there were twice as many *susceptible* people, how would this affect the number of people who *become infected*?

So,

$$S'(t) \text{is proportional to}$$

–If everything else was the same except there were twice as many *infected* people, how would this affect the number of people who *become infected*?

So,

$$S'(t) \text{is proportional to}$$

These combine to give

$$S'(t) =$$

What about $I'(t)$? It should equal
(number of people who get infected per day) − (number of people who ______________________).

So,

$$I'(t) =$$

Using Euler's Method on the SIR model

Suppose we're given:
$S' = -.00001SI$
$I' = .00001SI - I/14$
$R' = I/14$

with initial values at time $t = 0$ (in units of days):
$S(0) = 45400$, $I(0) = 2100$, $R(0) = 2500$.

(a) Find $S'(0)$, $I'(0)$, and $R'(0)$.

(b) Estimate $S(1)$, $I(1)$, $R(1)$. (First write down algebraic expressions for these quantities in terms of $S(0)$, $S'(0)$, $I(0)$, $I'(0)$, $R(0)$, $R'(0)$ and Δt and then plug in the numbers)

(c) Using Euler's Method with $\Delta t = 1$, repeat parts (a) and (b) above to find the number of infected people on the fourth day ($t = 4$).
Let's confirm our calculations of how S, I and R change with time over the space of 4 days by approximating the solution of the model by using Euler's Method with $\Delta t = 1$ day

t	S	I	R	Δt	$S'(t)$	$I'(t)$	$R'(t)$	ΔS	ΔI	ΔR