Warm-Up

1. Suppose you are given the following rate equation and initial condition:

$$H'(t) = -.008 \cdot (H(t) - 3.5), \qquad H(0) = 84.98.$$

   Using the "using slopes" method, estimate the value of $H(t)$ at $t = 2$ and $t = 4$.

2. In complete English sentences, explain as clearly as you can the *process* you are using to estimate these values.

Euler's Method

Suppose you have an initial value problem consisting of one rate equation and an associated initial value. In general, such initial value problems have the form

$$y'(t) = F(t, y(t)), \qquad y(t_0) = y_0$$

When you apply the "using slopes" method to find a piecewise linear function approximating the solution of this initial value problem, you are using a method originally developed by Euler, one of the early contributers to Calculus. Mathematically, Euler's Method may be described as follows:

Choose a stepsize Δt and the number n of steps you are going to take.

For each $k = 0, 1, 2..., n - 1$,

  Calculate $\Delta y \approx y'(t_k) \cdot \Delta t$, where $y'(t_k) = F(t_k, y(t_k))$.

  Calculate $y(t_{k+1}) = y(t_k) + \Delta y$.

  Set $t_{k+1} = t_k + \Delta t$.

The piecewise linear approximation then consists of line segments connecting the points

$$(t_0, y_0), \quad (t_1, y_1), \quad \dots \quad (t_n, y_n).$$