## Mathematical Modeling and Constant Rate Equations

Today we will introduce the general process of mathematical modeling, and then consider that process in the case of a particular example.

**A model is not an exact representation of a real phenomenon.**

**Purpose**: Why construct a model?

**Assumptions**: What is being simplified and omitted?

**Validation**: How do you know if the model is satisfactory?

### Modeling with Rates

It is often easier to construct a model involving *rates of change* of a variable than to model the variable directly. This observation is the basis of many applications of differential calculus to science.

1. Suppose a car starts from Chicago and travels towards Eagle Rock at a constant *rate* of 55 mph. Sketch a graph of its velocity, $v(t)$, as a function of elapsed time $t$ (hours).

2. For the same car, sketch a graph of the car's position, $s(t)$, (measured in miles travelled from Chicago) as a function of elapsed time $t$. What is the *slope* of this graph?

**Solutions to Constant Rate Equations**

By definition, *velocity* is the rate of change of *position* with time. Because of this, the notation $s'(t)$ is often used instead of $v(t)$.

3. Consider the rate equation $s'(t) = 55$. Can you find a solution to this equation? Is there more than one?

4. Suppose you are told that $s'(t) = 55$, *and* that $s = 25$ when $t = 0$. The second part of this statement is called an "initial condition," and is more commonly be written with the notation $s(0) = 25$. A rate equation together with an initial condition is called an *initial value problem*. Can you solve this initial value problem?

5. Once again let $s(t)$ denoting the car's distance from Chicago at elapsed time $t$. Write an English sentence explaining what the initial value problem is describing with this interpretation of $s(t)$.