Definition (**Proportionality**)

A variable quantity w is *(directly) proportional* to a variable quantity z if there is a *constant* c such that

$$w = c\,z.$$

Examples:

The volume, V, of a fixed mass of an ideal gas is directly proportional to its temperature, T, if the pressure is constant. *(Charles' Law)*

The magnitude F of the force exerted by a moderately stretched spring is directly proportional to the change ΔL in its length. *(Hooke's Law)*

2. Suppose a skydiver (who has already jumped out of an airplane) drops a ball. Is the velocity v of the ball directly proportional to the time t which has elapsed since the ball was dropped? Is the *change in velocity*, Δv, directly proportional to the *change in elapsed time*, Δt?

Elapsed Time, t	(sec):	0	1	2	3	4
Change in Elapsed Time, Δt						
Velocity, v	(ft/sec):	37	69	101	133	165
Change in Velocity, Δv						

Definition (**Linear Function**)

$y = f(x)$ is the rule of a *linear function* if a *change* in the input variable x yields a *proportional change* in the output variable y.

Thus, if x_1 and x_2 are input values, and $y_1 = f(x_1)$ and $y_2 = f(x_2)$ are corresponding output values, then

$$y_2 - y_1 = k\,(x_2 - x_1), \quad \text{for some constant } k.$$

The rule of a linear function f can be given in *slope-intercept* form as

$$y = mx + b, \qquad \text{slope } m \text{ and '}y\text{'-intercept } b,$$

or in *point-slope* form as

$$y_0 = f(x_0), \quad \Delta y = m\Delta x, \qquad \text{slope } m.$$

Notation: $\Delta y = y - y_0$ and $\Delta x = x - x_0$.

3. Starting with the point-slope form, derive the slope-intercept form.

4. Is $y = p(x) = x$ linear? Is $w = q(z) = 3z + 5$ linear? Why?