

Name \_\_\_\_\_

**Triangular Numbers**

The numbers 1, 3, 6,... are called the first three *triangular numbers* since they may be represented by triangular patterns of dots.

- (a) Draw pictures for the first few triangular numbers here...
- (b) Make a table of the first 10 triangular numbers and, for any  $n$ , give a formula for the  $n$ -th triangular number. Call the  $n^{\text{th}}$  triangular number  $T_n$ . To prove your formula, try to give a picture with dots which illustrates your result. (Notice that what you have actually found is a formula for the sum of the first  $n$  natural numbers.)
- (c) What is the sum of any two consecutive triangular numbers? That is find a formula for  $T_n + T_{n+1}$ . Prove your answer is correct, using algebra and/or your result from the previous question in (b). Now draw a picture with dots to illustrate this result.

(d) Prove that if  $T_n$  is a triangular number, then so is  $9T_n + 1$ .

(e) Explain why each number in the following sequence is a triangular number:

$$1, 10, 91, 820, \dots$$

**HINT:** you can think of the above sequence as...

$$1, 1 + 9, 1 + 9 + 81, \dots, 1 + 9 + 9^2 + \dots + 9^k, \dots$$