Name _____

**Poison Game Solution**

Write down (in complete sentences) the winning strategy for the Poison Game. Make sure to consider all cases.

A strategy is a way to play the game so that one or the other player emerges as winner every time the strategy is employed. The strategy for this game depends on the number of coins that are used.

Case 1: If there are $3k + 1$ coins (like 4, 7, 10, etc.) then the player who goes second will always win if they use this strategy:

**\*If player 1 takes 1 coin, the second takes two; if player 1 takes 2 coins, the second takes 1. This will eliminate coins in multiples of three leaving player 1 with poison.**

Case 2. If there are now $3k + 2$ coins (like 5, 8, 11, etc.) then the first player has a winning strategy by taking 1 coin to start and employing strategy \* on the second player (because $3k + 2 = 1 + 3k + 1$).

Case 3. Similarly, $3k$ coins (like 3, 6, 9, etc.) the first player has a winning strategy if he/she takes 2 coins and employs \* against the second player (because $3k = 2 + 3(k - 1) + 1$).

Let's say that there are two players but each can take from 1 to 3 coins each time, and the one left with a coin loses. What would be the winning strategy then?
What can we learn from the game above? The strategy \* above is used to get rid of coins in multiples of 3 in the basic Case 1. Here the strategy will be to get rid of them in multiples of 4.

Case 1. If there are $4k + 1$ coins, the second player will win employing the strategy

**\*\* Whatever player 1 takes, player 2 takes 4 - that number. This will eliminate coins in multiples of 4, leaving player 1 with poison.**

Case 2. If there are $4k + 2$ coins, player 1 takes 1 coin to start and uses strategy \*\* against player 2 and is victorious (because $4k + 2 = 1 + 4k + 1$).

Case 3. In the case of $4k + 3$ coins, player 1 takes 2 coins to start and uses \*\* to win.

Case 4. In the case of $4k$ coins player 1 takes 3 to start and uses \*\* to win.

Note: What I've described is a strategy to WIN (most of the time for player 1). Certainly both players can just play blindly and see what happens, but this gives a strategy to win.