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# Paleomagnetic Correlation of Columbia River Basalt Flows Using Secular Variation

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A statistical method is developed to evaluate stratigraphic correlations that are based on the similarity of paleomagnetic directions. It involves comparison of the probabilities that such directions could have arisen by either simultaneous or random samplings of the ancient magnetic field. To calculate these probabilities one must estimate the effects of paleomagnetic errors and paleosecular variation, respectively. The method is tested successfully on sections of 8-10 Grande Ronde Basalt (Columbia River Basalt Group) flows at Sentinel Gap and Umtanum Ridge, which are separated by only 25 km and contain two flows thought to be the same on geological grounds. More significantly, for widely separated sections at Sentinel Gap and Butler Canyon the analysis again shows that simultaneous magnetization is a much more likely explanation of the general similarity of paleomagnetic directions than chance agreement of ancient field directions. The simplest interpretation of this result is that many of the flows at the two sites are the same, implying that series of Grande Ronde flows extended over distances of several hundred kilometers.

## INTRODUCTION

Geomagnetic reversals have been the basis for most applications of paleomagnetism to stratigraphy. Because magnetic polarity is only a simple binary variable, it must be combined with other data in order to make reliable correlations. Thus *Swanson et al.* [1979] used magnetic polarity in combination with geologic and geochemical data to correlate sections of the Columbia River Basalt Group in the northwestern United States. Even so, the resolution of these correlations was limited; only in cases where the mineralogy or chemistry was distinctive could an individual flow be recognized and correlated within a group of flows of the same polarity. In addition, *Choiniere and Swanson* [1979] found a very unusual paleomagnetic direction at several widely separated sites on the Columbia River Plateau and the Oregon coast. The unusual direction, which presumably occurred during a reversal or excursion, helped strengthen a correlation which had been proposed on the basis of chemistry. However, one rarely has the opportunity to make correlations based on transitional field directions because of the relative infrequency and short duration of geomagnetic reversals and excursions.

Fortunately, for rocks which are reliable paleomagnetic recorders, there exists the possibility of more detailed correlations based on the smaller, more frequent secular variation of the geomagnetic field. In this way, otherwise similar geologic units have been distinguished by small differences in their paleomagnetic direction [e.g., *Hatherton,* 1954, and *Noble et al.,* 1968]. A more direct approach is to make stratigraphic correlations based on the similarity of paleomagnetic directions. Utilizing the fact that paleomagnetic direction is a precise parameter whose variation may be conveniently described by statistical models, *Cox* [1971] presented a method for estimating quantitatively the validity of such correlations and applied it to several large sheets of ash flow tuffs in New Zealand. *Grommé et al.* [1972] have used this method on tuff sheets in the western United States.

Correlation based on paleomagnetism in the sense described above has a rather special meaning. Consider, for example, two outcrops of basalt, say flow X and flow Y, which we have reason to suspect might be exposures of the same flow. Further, suppose that paleomagnetic directions from the two outcrops are found to be quite different from one another. Using the historic record of the geomagnetic field as a guide, the length of time for the field direction at a site to change by a paleomagnetically detectable amount is of the order of 10 to 100 years. Thus, to the extent that we can demonstrate that the difference in paleomagnetic direction between X and Y is reflecting changes in the ancient field direction rather than infidelity of the paleomagnetic recording process, experimental error, and spatial field variations, we can conclude that the flows sampled the ancient field at different times and are therefore not the same. One could say that the two flows represent different 'geomagnetic instants'. This term refers to periods of time, generally less than 100 years in length, which are short with respect to the time scale of geomagnetic secular variation.

Conversely, if the paleomagnetic directions from the two outcrops were quite similar, we would consider it more likely that flows X and Y had been erupted during the same geomagnetic instant (and thus could be the same flow). If the ancient field direction which the flows recorded was unusual, in the sense of being a direction which the field assumed infrequently, the case for simultaneity is improved. However, it is always possible that X and Y represent different geomagnetic instants when the field, by chance, happened to have very similar directions. The agreement of paleomagnetic directions in this case would be merely fortuitous. Clearly, if similar paleomagnetic directions are to be used as a basis for correlation, it is necessary to estimate the relative likelihood of both fortuitous and significant agreement in direction.

In this paper we first describe in general terms our statist-



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ical method for quantitative paleomagnetic correlation. Next we compare it with the method of *Cox* [1971] and discuss critical differences between the two. Finally, we apply our method to correlations between single flows and sequences of flows in the Grande Ronde Basalt on the Columbia River Plateau. A complete list of the symbols we use in the following discussion may be found in the notation section.

## STATISTICAL METHOD

For the purpose of stratigraphic correlation, there are two pure hypotheses which we can test relatively easily with paleomagnetic data. The first is that the paleomagnetic directions from two stratigraphic horizons represent 'random' samplings of the ancient field; i.e., the interval between horizons is longer than the characteristic time scale of secular variation. We will refer to this as the 'random hypothesis' ($H_r$). The second is that two paleomagnetic directions represent samplings of the same geomagnetic instant; i.e., the interval between horizons is short compared to the time scale of secular variation. We will refer to this as the 'simultaneous hypothesis' ($H_s$), bearing in mind that we mean simultaneity in this limited sense. There are, of course, any number of hypotheses between $H_r$ and $H_s$: for example, that two paleomagnetic directions represent samplings of the ancient field separated in time by 500 to 1000 years. To calculate the probability associated with such a hypothesis, however, requires more specific information about geomagnetic secular variation than is generally available.

The essence of the problem is to calculate the likelihood $P(D:H)$ that paleomagnetic data ($D$) could have arisen from the random or simultaneous hypothesis. The ratio of the likelihoods, $P(D:H_s) / P(D:H_r)$, expresses how likely the simultaneous hypothesis is compared to the random hypothesis in light of the paleomagnetic data. But usually we have information other than just the paleomagnetic data; in fact, often it is such information that is the motivation to evaluate a correlation. If we express this additional knowledge as a prior probability, $P(H)$, that either $H_s$ or $H_r$ is true, then we can use Bayes Theorem [e.g., *Phillips*, 1973] to find the posterior probability $P(H:D)$:

$$P(H:D) = \frac{P(H)P(D:H)}{P(D)} \tag{1a}$$

where

$$P(D) = P(H_s)\,P(D:H_s) + P(H_r)P(D:H_r) \tag{1b}$$

The posterior probability expresses the probability of $H_r$ or $H_s$, given both paleomagnetic data and the prior probability. $P(D)$, the total probability of $D$, serves as a normalizing factor.

### *Probability Distribution*

It has been shown that the time averaged geomagnetic field is fairly well approximated by a geocentric axial dipole [e.g., *McElhinny*, 1973; *McElhinny and Merrill*, 1975]. At any time, however, the field direction at a site will generally differ from the axial dipole direction due to nondipole and nonaxial dipolar components of the geomagnetic field. *Fisher* [1953] proposed a spherical distribution which has been widely used in paleomagnetism. In this distribution, the density of directions lying $\theta$ degrees from their mean is given by

$$P(\theta) = \frac{\kappa}{4\pi \sinh \kappa} \exp(\kappa \cos \theta) \tag{2}$$

where $\kappa$ is a precision parameter that is inversely related to the angular variance. In practice, the true value of $\kappa$ is never known and an estimate is used instead in (2). If we use $k$ to denote our estimate of the dispersion due to secular variation, then a large amount of secular variation will produce a distribution of field directions at a site that is characterized by small $k$. Low secular variation will be described by large values of $k$.

*Cox* [1970] and others have argued that the condition of axial symmetry is more nearly satisfied when field directions are transformed to virtual geomagnetic poles (VGP's). The non-Fisherian nature of varying field directions is especially pronounced at low latitudes, and in such cases it would be preferable to use VGP's in the statistical method that will be discussed below. However, our application of the method will involve the Columbia River basalt, which lies near 45° N latitude. Field directions at this latitude more closely satisfy the condition of axial symmetry, and therefore the assumption of Fisherian directions will not seriously affect our results. We will consider directions rather than VGP's simply to make the statistical method more direct and easily explainable.

If directions are thought of as vectors originating from the center of a unit sphere, then the Fisher distribution may be imagined as a probability density function on the surface of the sphere. The mean or 'expected' field direction is the pole about which small circles of equal probability lie. Any range of directions defines an area on the surface of the sphere, and the associated probability is just the integral of the Fisher density function over this area (Figure 1).

### *Probability Assuming Random Hypothesis*

We have data ($D$) in the form of two paleomagnetic directions $\vec{X}$ and $\vec{Y}$ from sites X and Y. We want to know

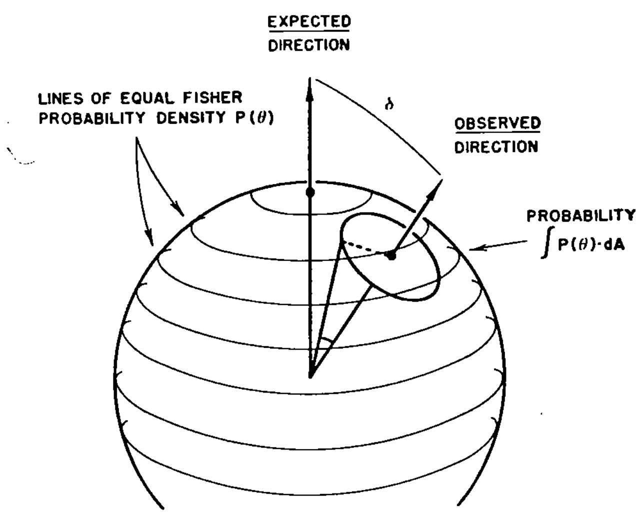


Fig. 1. Probability on a sphere. Fisher probability density function $P(\theta)$ is centered on expected direction, and observed direction lies at $\delta$ degrees. Probability associated with all directions within $\alpha$ degrees of observed direction is found by integrating $P(\theta)$ over the circular area shown.

the likelihood $P(D:H_r)$ that data in at least as good agreement as $D$ could arise by coincidence. Assume that $\vec{X}$ and $\vec{Y}$ are independent samples of the distribution of paleomagnetic field directions, i.e., that the two sites were magnetized at very different times relative to the time scale of secular variation and that each acquired its magnetization within an interval short compared to that time scale. On the unit sphere described previously, consider the spherical cap centered about direction $\vec{X}$ with angular radius $\alpha$ just large enough to include a second direction $\vec{Y}$ (Figure 2*a*). The probability $P_{XY}$ associated with the spherical cap may be thought of as the chance of site Y recording an ancient field direction within $\alpha$ degrees of the direction recorded by an earlier site X. As expected, the value of $P_{XY}$ lessens when $\alpha$ is decreased or when $\delta$, the angular distance from the expected axial dipole direction, is increased. The probability will also decrease with increasing $k$, the precision parameter which describes the degree of secular variation appropriate for the particular case.

In calculating $P_{XY}$, we have assumed that direction $\vec{X}$ came before direction $\vec{Y}$. It is equally possible, of course, that site Y preceded X. Thus we must also compute $P_{YX}$, the probability associated with the spherical cap centered on direction $\vec{Y}$ that just includes $\vec{X}$. The likelihood we seek must give equal weight to the two possibilities; that is,

$$P(D:H_r) = \tfrac{1}{2}(P_{XY} + P_{YX}) \tag{3}$$

*Probability Assuming Simultaneous Hypothesis*

Next we seek the likelihood $P(D:H_s)$ of the two sites having directions at least as different as $\vec{X}$ and $\vec{Y}$, assuming that they were magnetized during the same geomagnetic instant. There are many factors which will cause two such sites to yield different paleomagnetic results. These include orientation errors during sampling, measurement errors, uncertainty in correcting for rotation of beds, incompletely removed secondary components of magnetization, and actual differences in the ancient field direction between the two sites. Clearly, the more distinct the paleomagnetic directions from the two sites, and the smaller the errors such as listed above, the less likely the simultaneous hypothesis will be.

For each of the two sites the magnitude of explainable errors can be expressed in terms of angular standard deviation $\bar{s}$ of the site mean about the ancient field direction. (Note that a bar over a statistical parameter will be used to indicate that it refers to a population of mean directions rather than individual directions.) This can in turn be converted to $\bar{k}$, the estimate of $\kappa$ of the corresponding Fisher distribution, by

$$\bar{k} = \frac{\ln(0.37)}{\cos \bar{s} - 1} \tag{4a}$$

([*Watson and Irving*, 1957], $\bar{k} > 3$), which is approximated to within about 0.5% when $\bar{s} \leq 20°$ by the very simple formula

$$\bar{k} = 2 / \bar{s}^{\,2} \tag{4b}$$

([*Cox*, 1970]; $\bar{s}$ in radians). In this case, the precision parameter does not reflect the amount of secular variation but rather the scatter inherent to the paleomagnetic recording and measuring process. The value of $\bar{k}$ will depend in part on local conditions, and hence it will generally differ even for two sites X and Y of the same horizon. Thus the problem of determining $P(D:H_s)$ is equivalent to the following: if $\vec{X}$ and $\vec{Y}$ are directions drawn from Fisher distributions about the same mean but with different precision parameters (i.e., $\bar{k}_X$ and $\bar{k}_Y$), and $\alpha$ is the angle between

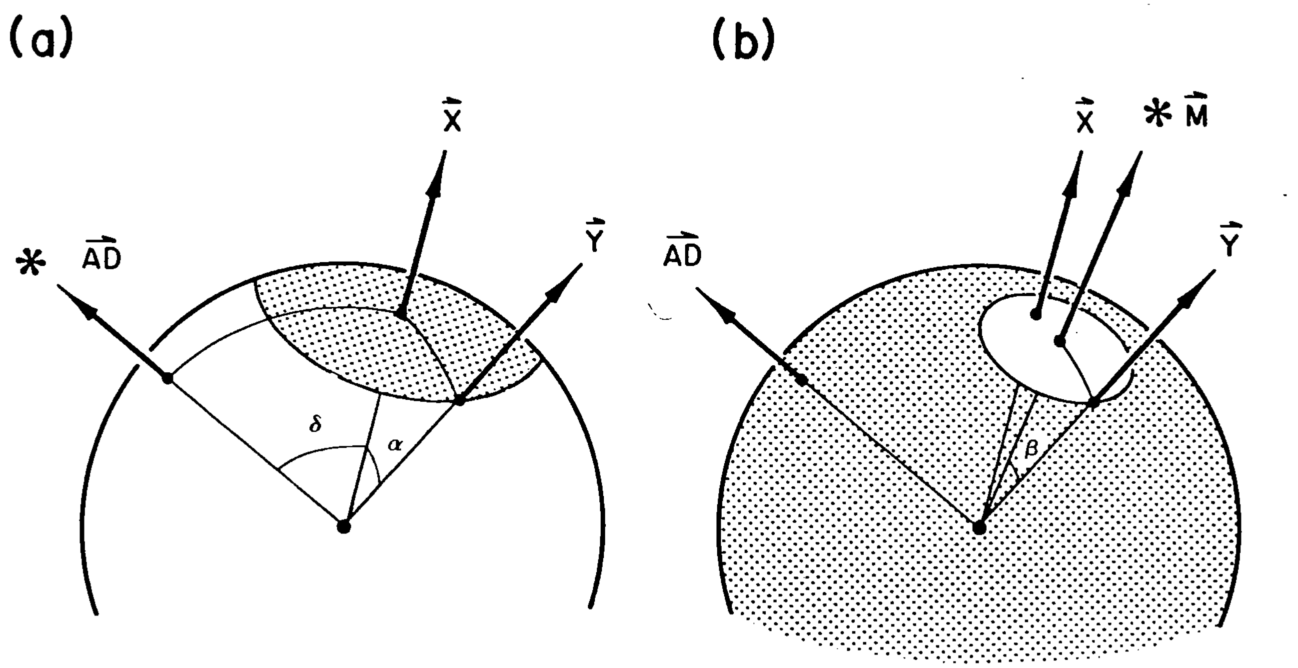


Fig. 2. (a) Circular area of integration for $P(D:H_r)$. The center (denoted by an asterisk) of the Fisher probability density function is the axial dipole direction. Integral of $P(\theta)$ over the shaded area gives the probability associated with all directions at least as close to $\vec{X}$ as $\vec{Y}$. (b) Circular area of integration for $P(D:H_s)$. The center (*) of $P(\theta)$ is the mean direction $\vec{M}$. Integral of $P(\theta)$ over the shaded area gives the probability associated with all directions at least as far from $\vec{M}$ as $\vec{Y}$.

them, what is the probability distribution function for $\alpha$? This approach has the advantage that we need not know the ancient field direction, but it involves a numerically cumbersome double integration for each pair of values $\bar{k}_X$ and $\bar{k}_Y$. Therefore we have developed an approximate but much simpler method which requires that we estimate the ancient field direction.

Without a priori knowledge of the ancient field, the best estimate of the ancient field direction from two sites belonging to the same horizon is a weighted vector mean of their respective paleomagnetic directions. Assume that this mean direction is the actual direction about which the average directions from the sites are scattered due to the errors mentioned above. Using a value of $\bar{k}$ which reflects the possible errors in paleomagnetic direction $\vec{Y}$, it is simple to calculate the probability $P_Y'$ associated with the spherical cap centered on the mean paleomagnetic direction with angular radius $\beta_Y$ which is just large enough to include $\vec{Y}$ (Figure 2*b*). Direct integration of (2) yields

$$P_Y' = \frac{\exp(\bar{k}_Y) - \exp(\bar{k}_Y \cos\beta_Y)}{2\sinh \bar{k}_Y} \qquad (5)$$

The probability outside the circular area, $P_Y = (1 - P_Y')$, is that associated with all paleomagnetic directions at least as far away as $\vec{Y}$ from the estimated ancient field direction. For $\bar{k}_Y \geq 3$ so that $\exp(-2\bar{k}_Y)$ can be neglected compared to 1, we have the simple result

$$P_Y = \exp[\bar{k}_Y(\cos\beta_Y - 1)] \qquad (6)$$

A similar calculation can be performed for direction $\vec{X}$. A different value of $\bar{k}$ may be appropriate if, for instance, flow X was a less reliable paleomagnetic recorder than Y. In general, the size of the spherical cap of integration will be different also since the estimated ancient field direction will be weighted toward the more precisely determined of $\vec{X}$ and $\vec{Y}$. The product, $P_X P_Y$, is the probability of flows X and Y being magnetized during the same geomagnetic instant, yet yielding paleomagnetic directions at least as different as they do:

$$P(D:H_s) = \exp[\bar{k}_X(\cos\beta_X - 1) + \bar{k}_Y(\cos\beta_Y - 1)] \qquad (7)$$

This probability will decrease as $\vec{X}$ and $\vec{Y}$ become more dissimilar and as the values of $\bar{k}_X$ and $\bar{k}_Y$, which are inversely related to the amount of error that can be accounted for, get large.

How good an estimate of $P(D:H_s)$ is given by this procedure can be checked by the method of *Watson* [1956]. His method employs the F test to evaluate the probability that two populations have the same mean, but it can only be used in cases where the populations have the same precision parameter. It has the great advantage of not requiring an estimate of the true mean direction and thus, when it can be applied, gives the best assessment of $P(D:H_s)$. However, the essence of the correlation problem is comparing estimates of mean directions for cases where the sources and magnitudes of errors for each estimate may be quite different. Since Watson's statistic tests the indistinguishability of two populations (both mean and variance), it is generally inappropriate when correlating distinct geologic units.

Nevertheless, we can check our estimate of $P(D:H_s)$ against Watson's F statistic by restricting our attention to the special case of equal precision parameters. As shown in Appendix B, the two methods agree quite well over the range of parameters likely to be encountered in paleomagnetic studies. Our method of estimating $P(D:H_s)$ (equation (7)), which is well suited and easily applied to the general problem of geologic correlation where unequal precision parameters are usually encountered, is the best we know of.

### *Relative Probability*

The ratio of the likelihoods is a measure of the relative probability of the random and simultaneous hypotheses in light of the paleomagnetic data. For instance, $P(D:H_s)/P(D:H_r) = 9$ signifies that the paleomagnetic data support $H_s$ 9 times more strongly than $H_r$. As mentioned earlier, however, it is conceptually neater and often more useful to use Bayes theorem (1*a*) to compute the posterior probabilities $P(H_s:D)$ and $P(H_r:D)$. This enables us to include other information we may have in our assessment of the proposed stratigraphic correlation by expressing it in the prior probabilities. If we take the prior probabilities as equal, say $P(H_r) = P(H_s) = 0.5$, then the posterior probabilities reflect only the paleomagnetic data.

It is important to see that the values of $P(H_r:D)$ and $P(H_s:D)$ that result from the application of Bayes theorem give the relative probability of $H_r$ and $H_s$. A high value of $P(H_s:D)$ only means that $H_s$ appears likely when compared to $H_r$. This results directly from the normalization by $P(D)$. A third hypothesis, intermediate between $H_r$ and $H_s$, may be much more likely than either $H_r$ or $H_s$. However, the real purpose of the calculation is to quantify how the similar paleomagnetic directions affect our confidence in a stratigraphic correlation of the sites. The relative probabilities of $H_r$ and $H_s$ provide the most direct indication of how likely it is that two sites are closely related in time.

### *Comparison With a Similar Method*

Before considering the application of these probability calculations to stratigraphic correlation, it is instructive to compare the method described above to that of *Cox* [1971]. Having obtained nearly identical paleomagnetic directions from several widely separated exposures of ignimbrite in New Zealand, Cox performed a calculation similar to that described here for $P(D:H_r)$. He took as his starting point the usual paleomagnetic convention that two directions $\vec{X}$ and $\vec{Y}$ with overlapping $\alpha_{95}$ [*Fisher*, 1953] circles are indistinguishable, assumed that the distribution of field directions was Fisherian, and chose a value of $k$ to describe the secular variation at the latitude of New Zealand. The probability calculated by Cox (his equation (3), with a typographical error of $1/2\pi$ omitted)

$$P = k[1 - \cos(\alpha_{95,X} + \alpha_{95,Y})] \exp k(\cos\delta_X - 1) \qquad (8)$$

is an approximation. Since $\delta_X$ is the angle between $\vec{X}$ and the axial dipole direction, this equation is just the Fisher density function at $\vec{X}$ multiplied times the area of the spherical cap centered on $\vec{X}$ with angular radius equal to the sum of the $\alpha_{95}$ values for $\vec{X}$ and $\vec{Y}$. For the approximation to be valid, the angular radius must be small. Thus, Cox's statistic is an estimate of the probability that two randomly sampled field directions would be close enough for their $\alpha_{95}$ circles to overlap. The probability that the two directions were recorded simultaneously is not considered.

We believe that our method is conceptually the more correct for several reasons. For instance, similar paleomagnetic directions will result in the probability of $H_r$ being quite low. However, if we have a great deal of confidence in the paleomagnetic results, the difference in the directions, though slight, may reduce our belief in $H_s$ a great deal also. The relative probabilities of $H_r$ and $H_s$ will be much more informative than just the probability of the former.

There are also problems in using the overlap of circles of confidence as a criterion of indistinguishability. Most importantly, non-overlap of the $\alpha_{95}$ circles does not exclude the possibility that two sites were magnetized simultaneously. A variety of effects can cause the mean direction of magnetization to differ slightly from site to site in the same cooling unit. Since $\alpha_{95}$ is proportional to $1/\sqrt{N}$, circles of confidence for simultaneously magnetized sites will not overlap if the number of samples $N$ is large enough. For these and other reasons, it is important to calculate explicitly a probability for the simultaneous hypothesis, using both the within-flow scatter and other information as illustrated by the following section, and to compare this with the probability for the random hypothesis.

Thus, although the goals and the physical models are similar for the two methods, the predicted probabilities differ fundamentally in some respects. Perhaps this is seen most clearly by considering the effect of increasing the within-flow scatter, and therefore the $\alpha_{95}$'s and $\bar{k}$'s. In Cox's model the probability of the random hypothesis increases whereas in our model the simultaneous hypothesis becomes more likely.

## Application To Columbia River Basalt Stratigraphy

### *Sentinel Gap-Umtanum Ridge*

*Paleomagnetic directions and uncertainty.* With the goal of stratigraphic correlation in mind, we measured the paleomagnetic directions from a sequence of 12 flows of the Grande Ronde Basalt (the Lower Yakima Basalt of *Wright et al.* [1973]) in south-central Washington. Though generally flat-lying, the flows are well-exposed in an east-west striking anticline which has been transected by the Columbia River at Sentinel Gap (Figure 3). We drilled samples from roadcut and natural cliff exposures and oriented the cores using a magnetic compass. Partial demagnetization of samples in alternating fields (AF) of 100 to 200 Oe was sufficient to reduce secondary components of magnetization. The details of this straightforward paleomagnetic study have been reported by *Coe et al.* [1978].

The paleomagnetic directions after AF cleaning for the Grande Ronde flows at Sentinel Gap are summarized in Table 1, and the upper 10 flows have been plotted in sequence on an equal area projection in Figure 4. We have corrected the directions of magnetization for the tectonic dip of the flows. Although these attitudes were difficult to estimate in the field because of unfavorable exposure, they never appeared to be more than 11°. The individual flow designations are those of *Taylor* [1976].

Flow A, which lies at the bottom of the section, is reversely magnetized. Flow B and flows E through J yielded a tightly clustered group of directions centered about the axial dipole direction for Sentinel Gap ($I = 65°$). The

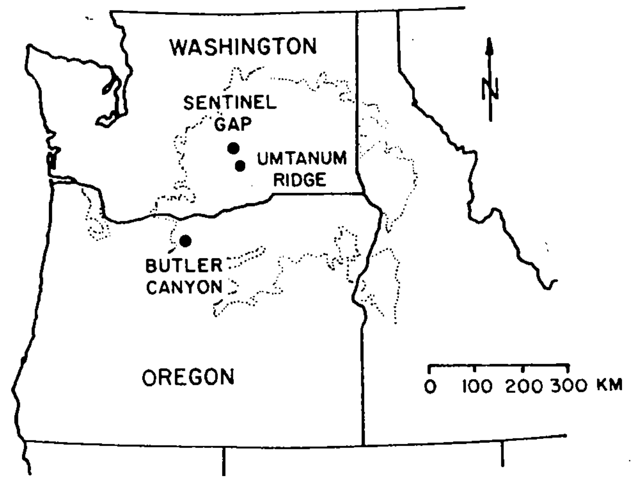


Fig. 3. Location map showing sites mentioned in text. Dotted line indicates outcrop extent of Columbia River basalt.

directions from flows C and D lie close to one another, but are nearly 25° shallow and westerly with respect to the axial dipole direction. The two flows which cap the section, the Rocky Coulee and Museum flows, form a third group of directions which are steep and westerly, lying 15° from the expected field direction. The clustered nature of the directions suggests that the volcanism was episodic, with pulses of eruption when flows were produced fairly quickly with respect to field variations. That is, the 10 flows above flow B have recorded, in sequence, shallow, moderate, and steep field directions.

*Beck et al.* [1978] have sampled a sequence of eight flows of the Grande Ronde Basalt at Umtanum Ridge located 25 km to the southwest of Sentinel Gap in Washington (Figure 3). Their results are reproduced in Table 1 and the directions plotted in Figure 4. The correlation between the two sections on the basis of geological and geochemical evidence is strong; in particular, flows D and Rocky Coulee at Sentinel Gap are very likely the same as 4 and 11 at Umtanum Ridge. Thus the paleomagnetic data from these sections provides an excellent opportunity to test the statistical method outlined above.

Before we can do so, it is necessary to evaluate the errors which affect the mean direction of each flow. This is crucial for a valid estimation of the probability of the simultaneous hypothesis. Uncertainty in the flow-averaged directions may arise from many factors: orientation and measurement errors, unremoved components of secondary magnetization, anisotropy of acquisition of TRM, and magnetic anomalies of various scales. On the level of a single flow, these factors may cause both random and systematic errors. Provided that each flow is sampled sufficiently, the magnitude of the random errors is well estimated by the sample-to-sample scatter. The statistic of relevance is $\bar{s}_w$, the within-flow angular standard deviation of the mean. This is easily calculated from the deviations of individual sample directions from the flow mean, and for most paleomagnetic data can be fairly well approximated using equation (4*b*) with

$$\bar{k}_w = Nk_w \tag{9}$$

where

$$k_w = \frac{N-1}{N-R} \tag{10}$$

TABLE 1. Paleomagnetic Data

| Flow | $N$ | $D$ | $I$ | $k_w$ | $\alpha_{95}$ | $\bar{s}_w$ | $\bar{s}_t$ | $\bar{s}_f$ | $\bar{s}$ | $\bar{k}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | *Sentinel Gap* | | | | | |
| Museum | 7 | 344.3 | 75.9 | 1126 | 1.8 | 0.9 | 3.5 | 0.5 | 3.7 | 480 |
| Rocky Coulee | 7 | 332.5 | 80.4 | 241 | 3.9 | 2.0 | 3.5 | 0.5 | 4.1 | 391 |
| J | 7 | 2.6 | 60.8 | 298 | 3.5 | 1.8 | 3.5 | 0.5 | 4.0 | 410 |
| I | 7 | 0.5 | 64.7 | 117 | 5.6 | 2.8 | 3.5 | 0.5 | 4.5 | 324 |
| H | 5 | 22.5 | 72.8 | 697 | 2.9 | 1.4 | 3.5 | 0.5 | 3.8 | 455 |
| G | 7 | 1.3 | 56.5 | 540 | 2.6 | 1.3 | 3.5 | 0.5 | 3.8 | 455 |
| F | 7 | 7.1 | 59.4 | 1126 | 1.8 | 0.9 | 3.5 | 0.5 | 3.6 | 507 |
| E | 7 | 355.0 | 69.0 | 1262 | 1.7 | 0.9 | 4.5 | 0.5 | 4.6 | 310 |
| D | 7 | 349.3 | 41.8 | 45.9 | 9.0 | 4.5 | 4.5 | 0.5 | 6.4 | 160 |
| C | 7 | 337.1 | 36.9 | 38.9 | 9.8 | 4.9 | 4.5 | 0.5 | 6.7 | 146 |
| (B) | 7 | 359.4 | 67.1 | 754 | 2.2 | | | | | |
| (A) | 7 | 267.3 | −86.8 | 117 | 5.6 | not used for correlation | | | | |
| | | | | | *Umtanum Ridge* | | | | | |
| 11 | 7 | 333.3 | 84.0 | 28.5 | 11.5 | 5.7 | 3.5 | 0.5 | 6.7 | 146 |
| 10 | 9 | 6.6 | 57.0 | 103 | 5.1 | 2.7 | 3.5 | 0.5 | 4.4 | 339 |
| 9 | 7 | 4.3 | 68.9 | 159 | 4.8 | 2.4 | 3.5 | 0.5 | 4.3 | 355 |
| 8 | 6 | 7.3 | 71.5 | 29.7 | 12.5 | 6.1 | 3.5 | 0.5 | 7.1 | 130 |
| 7 | 8 | 359.1 | 64.5 | 203 | 3.9 | 2.0 | 3.5 | 0.5 | 4.1 | 391 |
| 6 | 8 | 344.7 | 70.2 | 89.1 | 5.9 | 3.0 | 3.5 | 0.5 | 4.6 | 310 |
| 5 | 7 | 355.3 | 64.7 | 45.9 | 9.0 | 4.5 | 3.5 | 0.5 | 5.7 | 202 |
| 4 | 7 | 344.2 | 39.6 | 98.9 | 6.1 | 3.1 | 3.5 | 0.5 | 4.7 | 297 |
| | | | | | *Butler Canyon* | | | | | |
| 2 | 2 | 306.0 | 71.0 | 166 | 19.5 | 4.4 | 4.0 | 1.0 | 6.0 | 181 |
| 3 | 2 | 324.8 | 73.3 | 499 | 11.2 | 2.6 | 4.0 | 1.0 | 4.9 | 276 |
| 4 | 2 | 6.2 | 59.0 | 1001 | 7.9 | 1.8 | 4.0 | 1.0 | 4.5 | 324 |
| 5 | 2 | 351.8 | 55.1 | 334 | 13.7 | 3.1 | 4.0 | 1.0 | 5.2 | 247 |
| 6 | 2 | 9.5 | 58.4 | 125 | 22.5 | 5.2 | 4.0 | 1.0 | 6.6 | 149 |
| 7 | 2 | 4.1 | 57.5 | 499 | 11.2 | 2.6 | 4.0 | 1.0 | 4.9 | 276 |
| 8 | 2 | 7.0 | 58.5 | >1991 | <5.6 | 1.3 | 4.0 | 1.0 | 4.3 | 351 |
| 9 | 2 | 348.0 | 30.0 | 334 | 13.7 | 3.1 | 4.0 | 1.0 | 5.2 | 247 |
| 10 | 2 | 346.0 | 37.5 | 14.7 | 70.7 | 14.9 | 4.0 | 1.0 | 15.5 | 27.5 |

$N$ is the number of sample directions included in the mean paleomagnetic direction for each flow; $D$ and $I$ are the eastward declination and downward inclination of the mean direciton; $k_w$ is the precision parameter describing within-site dispersion, $\alpha_{95}$ is the angular interval of 95% confidence in the mean direction [*Fisher*, 1953]; $\bar{s}_w$, $\bar{s}_t$, $\bar{s}_f$, and $s$ are the angular standard deviations of the mean direction due to within-site dispersion, improper correction for the tilt of the flow, magnetic field anomalies, and the combined effect of the previous three; $\bar{k}$ is the precision parameter equivalent to $\bar{s}$.

[*McFadden*, 1980]. $R$ is the length of the resultant when $N$ sample directions are each given unit length and summed vectorially.

The systematic errors may often exceed the random errors, but they are much less straightforward to evaluate. Orientation and measurement errors for individual specimens may have a systematic bias, especially between different scientists and different pieces of equipment. We are not in a position to evaluate these errors and will neglect them on the assumption that they are generally outweighed by others. Unremoved secondary components of viscous remanent magnetization would also cause systematic errors. The angular error associated with such components will be greater the more unusual the direction of the field at the time the flow was magnetized. However, for two sites in the same flow the angular deviations of the mean directions are likely to be similar and thus may not have much effect on the correlation between the sites. If our AF demagnetization results at Sentinel Gap are representative of Umtanum Ridge as well, there are only a few flows for which the angular deviations themselves may be nonnegligible (flows C, D, 4, and 11). Bearing in mind that an underestimate of errors will lead to a lower estimate of the probability of the simultaneous hypothesis, i.e., a more conservative correlation, we will also neglect this source of error.

Magnetic anomalies are another potential source of errors in the flow mean directions. We may conveniently divide these anomalies into three categories arising from (1) the variations in magnetization and geometry of the flow in question and that which underlies it, (2) the magnetization of rocks in the general region of the site, and (3) the nondipole field generated in the core of the earth. The first two types have been discussed by *Doell and Cox* [1963] with regard to paleomagnetic errors in Hawaiian lava flows. The first we expect to cause mainly within-site (random) errors and the second between-site or between-section (systematic) errors. Given the planar geometry and regular and not particularly strong intensity of NRM of these Columbia River flows, we estimate systematic variations in direction of only 0.5°. For localities as close to each other as Sentinel Gap and Umtanum Ridge the variation in field direction between sections due to the nondipole field is of the order of 0.1° or less. Thus, we assign to each section an angular standard deviation $\bar{s}_f = 0.5°$, which represents the probable contribution of field anomalies to the discrepancy in field direction between the two localities at a given time.

By far the largest systematic error is the uncertainty in the original orientation of each flow. At Sentinel Gap we estimated the strike and dip of the basalt flows and corrected the paleomagnetic directions by assuming that the flows were originally horizontal and had been rotated about their strike. There are two difficulties with this method.

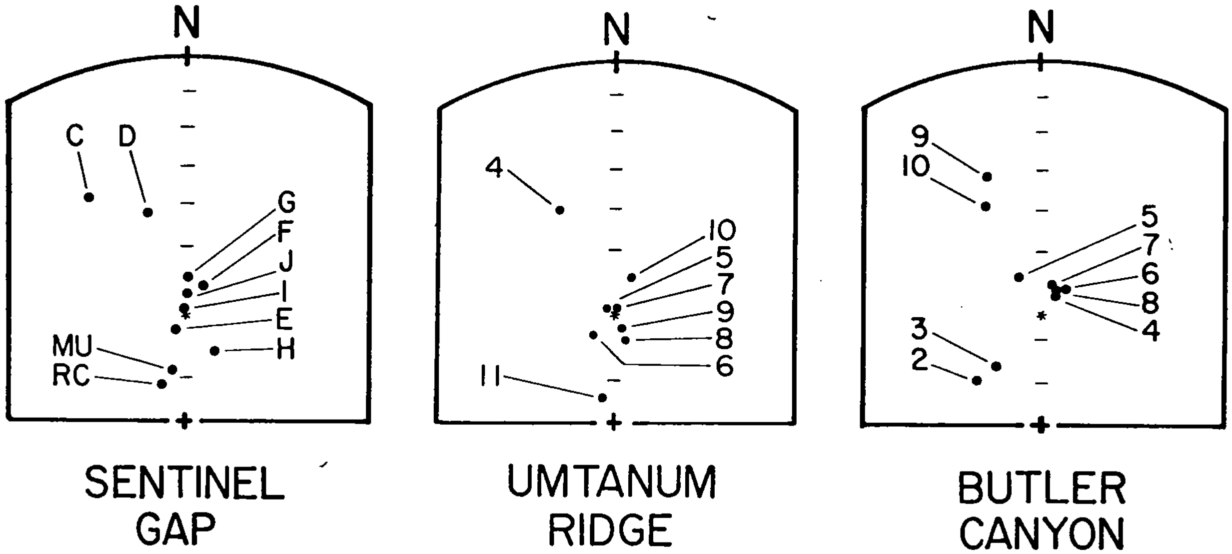


Fig. 4. Equal area plot of flow-averaged directions from Sentinel Gap, Umtanum Ridge and Butler Canyon. Center of net indicated by a plus sign, axial dipole direction by an asterisk. MU = Museum Flow, RC = Rocky Coulee Flow.

First, we have ignored any original slope that the paleotopography imposed on the flows. However, since the flows were evidently very fluid basalt and are quick thick, original dips of more than a few degrees would not be expected. We take 2.5° for the angular error introduced by original dip.

More troublesome is the uncertainty in our estimate of the dips. Flows A through E were sampled near the hinge of a large asymmetric anticline where dips changed from 10°-11° on the southern limb to nearly vertical on the northern limb. Although the northern limb has been largely removed through erosion, it appears that the folding was very tight. Dips in the hinge region must have changed dramatically over short distances. In addition, flows C, D and E, where sampled, were exposed nearly along their strike, making it even more difficult to determine their attitude with great accuracy. For flows C, D and E, we estimate the uncertainty in the attitude to be 3°-4°. Values from 2°-3° are more appropriate for flows higher in the section. Combining these uncertainties with that of original dip, we obtain estimates of 3.5° and 4.5°, respectively, for the total tectonic error $\bar{s}_t$ at Sentinel Gap. At Umtanum Ridge *Beck et al.* [1978] did not make tectonic corrections since dips measured in the field were less than 5°. Thus a rough estimate of the error is half this value, or 2.5°. Combined with the 2.5° uncertainty in original dip, $\bar{s}_t = 3.5°$ for Umtanum Ridge.

In summary, systematic errors introduced during sampling and measurement and due to unremoved secondary components are difficult to estimate but probably are quite small. For the example of Sentinel Gap and Umtanum Ridge, we will consider the within-site error which is easily and directly calculated, the estimated error introduced by correcting for the tilt of the flows, and a small correction for possible field anomalies. Because these three sources of error are independent, the total angular standard deviation is

$$\bar{s} = \left[ \bar{s}_w^2 + \bar{s}_t^2 + \bar{s}_f^2 \right]^{1/2} \qquad (11)$$

The values of $\bar{s}$ we obtain are summarized in Table 1 and range from 3.6° to 6.7° for the two sections. The corresponding $k$ ranges from 507 to 146. It is clear from Table 1 that $\bar{s}_w$ and $\bar{s}_t$ are the major components of $\bar{s}$ in our study. We include $\bar{s}_f$ primarily for completeness. However, in cases where the rocks being compared are farther apart or more intensely magnetic, $\bar{s}_f$ may make a significant contribution to the total error and should be considered carefully.

*Estimation of paleosecular variation.* In order to calculate $P(D:H_r)$ we must estimate the secular variation displayed by the ancient field which magnetized the Columbia River basalt flows. *McElhinny and Merrill* [1975] compiled paleomagnetic results from the last 5 m.y. and determined the variation of angular dispersion as a function of latitude. The authors considered instances where at least 15 spot readings of the ancient field from one site had been measured and averaged the angular dispersion of these sites over latitude strips 15° wide.

The curves in Figure 5 are interpolated from *McElhinny and Merrill*'s [1975] Table 5 and transformed using their equations (9) and (11). Thus we have converted VGP dispersion to dispersion of field direction and expressed this scatter using the precision parameter $k$. The results for low latitudes depend somewhat on whether it is assumed that field directions or VGP's are distributed in a Fisherian manner. As mentioned before, VGP's of present day field directions meet the conditions of azimuthal symmetry more closely than the directions themselves. However, the curves in Figure 5 demonstrate that at the latitude of Sentinel Gap (45 ° N), the difference is very small and that field directions are very nearly Fisherian. Reading the lower curve on Figure 5, it can be seen that a value of $k = 40$ describes the secular variation over the last 5 m.y. at the latitude of Sentinel Gap.

A more direct method is to calculate the total angular dispersion of the paleomagnetic directions from *Watkins and Baksi*'s [1974] extensive work in the Columbia River Basalt Group. This will give an estimate of secular variation appropriate for the Columbia River region rather than a worldwide average for latitude 45° N. In addition, use of this local estimate frees one from assuming that the geomag-

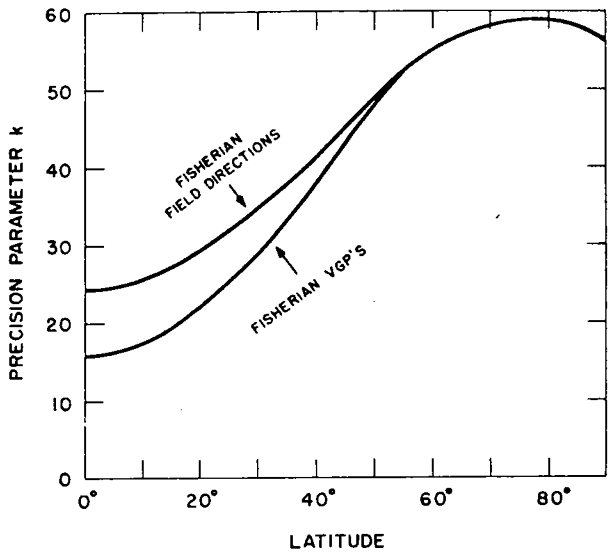


Fig. 5. Plot of dispersion of field directions ($k$) due to secular variation as function of latitude. Based on *McElhinny and Merrill*'s [1975] compilation of paleomagnetic results for last 5 m.y. At low latitudes, the value of $k$ depends on whether directions or VGP's are assumed to be distributed in a Fisherian manner.

netic field 15 m.y. ago behaved like the field has on the average for the last 5 m.y.

Using normally magnetized lavas from four sites which, according to the stratigraphic correlations of Watkins and Baksi, comprise nonoverlapping sequences spanning approximately 3 m.y., we obtain a value of $k = 30.1$. The contribution to the angular dispersion from experimental errors, which is reflected in the angular dispersion of sample directions, is taken to be 10° on the average [*McElhinny and Merrill*, 1975], and has been subtracted from the total angular dispersion.

The value of $k$ that we obtain indicates somewhat more variation in the field direction than is expected from *McElhinny and Merrill*'s [1975] worldwide averages. Field directions far from the axial dipole direction are less unexpected, and less distinctive as stratigraphic markers if secular variation is large. The use of the lower value of $k = 30.1$ will thus lead to a conservative estimate of the reliability of the correlations.

*Correlation between individual flows.* The two pairs of flows that we believe are the same from geological evidence allow us to illustrate and 'calibrate' the method. Consider first the correlation of flow D at Sentinel Gap with flow 4 at Umtanum Ridge. What is the likelihood $P(D:H_r)$ associated with the random hypothesis?

As can be seen in Table 2, the direction of flow D lies $\delta = 23.8°$ from the axial dipole direction and $\alpha = 4.4°$ from the direction of flow 4. To find the chance of the field lying as close to the flow D direction as does the flow 4 direction, assuming the two flows are independent samples, we must integrate the appropriate probability density function (2) over the spherical cap with angular radius $\alpha = 4.4°$ centered on the direction of flow D (Figure 1 and 2*a*). A value of $k = 30.1$ was chosen above as best for the Fisher distribution representing secular variation of field directions during the eruption of the Columbia River Basalt Group. Numerical integration yields the result $P_{D4} = 0.007$. This result could also be estimated from the probability curves provided in Appendix A.

Similarly, we find that the probability of the field lying within 4.4° of flow 4 is $P_{4D} = 0.004$. Then, as described earlier (3), $P(D:H_r)$ is the average of $P_{D4}$ and $P_{4D}$, that is, 0.006 (Table 2). The same calculation performed for the Rocky Coulee flow and flow 11 yields $P(D:H_r) = 0.013$ (Table 2).

Even though the likelihood associated with the random hypothesis is very small for both of these pairs of flows, this only takes on significance with regard to correlation when compared with the likelihood of the simultaneous hypothesis. $P(D:H_s)$ is easily calculated by (7); $\beta$ refers to the angle between the flow direction and the weighted mean direction of the two flows, the latter being the best estimate of the ancient field direction (Figure 2*b*). The correct weighting factor to use for each flow is $\bar{k}$, the precision parameter which reflects the error estimated for the flow mean direction, because $\bar{k}$ is proportional to the inverse variance (4b). A simple way to obtain $\beta$ is to use the angle $\alpha$ between flow directions that is given in Table 2. For instance, for the D-4 correlation

$$\beta_D = \frac{\bar{k}_4 \, \alpha_{D4}}{\bar{k}_4 + \bar{k}_D} = 2.9° \qquad (12)$$

Using (7), we find values of $P(D:H_s)$ of 0.729 for flows D and 4 and 0.810 for flows Rocky Coulee and 11.

To determine how sensitive $P(D:H_s)$ is to uncertainties in its input parameters, we varied the value of $\bar{k}$ by ± 10 % and $\beta$ by ± 0.1° for the D-4 and Rocky Coulee-11 correlations. In both cases, the range in $P(D:H_s)$ was approximately ± 6 %. Thus it should be kept in mind that the uncertainty in $P(D:H_s)$ is considerably greater than is indicated by the three significant figures we report in the tables.

$P(D:H_s)$ is about 120 and 60 times larger than $P(D:H_r)$ for the pairs D-4 and Rocky Coulee-11, respectively. This is a satisfying result, since there is every reason to believe we are comparing two outcrops of the same flow in each case. The relative probability of chance agreement is greater for the Rocky Coulee-11 pair simply because the directions from the flows are less unusual (that is, closer to the axial dipole direction) than the D-4 directions.

We may apply Bayes theorem (1*a*) to calculate the posterior probabilities, $P(H_r:D)$ and $P(H_s:D)$ by setting the prior probabilities $P(H_r)$ and $P(H_s)$ equal to 0.5. This is equivalent to using only the paleomagnetic data to choose between the hypotheses. We obtain $P(H_r:D) = 0.008$ and $P(H_s:D) = 0.992$ for the D-4 comparison, and 0.016 and 0.984 for Rocky Coulee-11. Naturally, since we used the same information as in the previous paragraph, we get the same result: $H_s$ is about 120 times as probable as $H_r$ for D-4 and 60 times as probable for Rocky Coulee-11. All we have done is normalize the previous probabilities so that they sum to 1. It is only when other information leads to assigning non-equal prior probabilities that the difference between $P(D:H)$ and $P(H:D)$ is not trivial.

The six remaining pairs of flows are characterized by more usual directions than the first two. Hence we would expect the probabilities associated with the random hypothesis to be higher and the problem of choosing between the two hypotheses to be more difficult. Actually, the probabilities are reasonably clear cut in five out of the six cases. Four of the pairs correlate well (E-5, H-8, I-9, and J-10), yielding ratios of $P(D:H_s) / P(D:H_r)$ that vary from 7.1 to 10.0. On the other hand, for flows F and 6 the

TABLE 2. Sentinel Gap-Umtanum Ridge Correlation (Flow by Flow)

| Flows | $\bar{k}$ | $\delta$ | $\alpha$ | $P(D:H_r)$ | $P(D:H_s)$ | Sequential Application of Bayes Theorem $P(H_r)$ | $P(H_s)$ | $P(H_r:D)$ | $P(H_s:D)$ |
|---|---|---|---|---|---|---|---|---|---|
| RC<br>11 | 391<br>146 | 17.2<br>20.0 | 3.6 | 0.013 | 0.810 | 0.500 | 0.500 | 0.016 | 0.984 |
| J<br>10 | 410<br>339 | 4.2<br>8.4 | 4.3 | 0.068 | 0.593 | 0.016 | 0.984 | 0.002 | 0.998 |
| I<br>9 | 324<br>355 | 0.2<br>4.4 | 4.5 | 0.086 | 0.607 | 0.002 | 0.998 | $2 \times 10^{-4}$ | >0.999 |
| H<br>8 | 455<br>130 | 11.3<br>7.2 | 4.8 | 0.069 | 0.691 | $2 \times 10^{-4}$ | >0.999 | $3 \times 10^{-5}$ | >0.999 |
| G<br>7 | 455<br>391 | 8.3<br>0.5 | 8.1 | 0.229 | 0.129 | $3 \times 10^{-5}$ | >0.999 | $5 \times 10^{-5}$ | >0.999 |
| F<br>6 | 507<br>310 | 6.3<br>7.9 | 14.2 | 0.522 | 0.003 | $5 \times 10^{-5}$ | >0.999 | 0.008 | 0.992 |
| E<br>5 | 310<br>202 | 4.6<br>2.0 | 4.3 | 0.077 | 0.708 | 0.008 | 0.992 | $9 \times 10^{-4}$ | 0.999 |
| D<br>4 | 160<br>325 | 23.8<br>26.8 | 4.4 | 0.006 | 0.729 | $9 \times 10^{-4}$ | 0.999 | $9 \times 10^{-6}$ | >0.999 |

The $\bar{k}$ is the precision parameter expressing the estimated uncertainty in the mean paleomagnetic direction for each flow; $\delta$ is the angular distance between the paleomagnetic and axial dipole directions; $\alpha$ is the angular distance between two paleomagnetic directions. $P(D:H_r)$ and $P(D:H_s)$ are the probabilities of similar paleomagnetic data ($D$) assuming the random and simultaneous hypotheses ($H_r$ and $H_s$); $P(H_r)$ and $P(H_r)$ are the prior (unconditional) probabilities of $H_r$ and $H_s$; $P(H_r:D)$ and $P(H_s:D)$ are the posterior probabilities of $H_r$ and $H_s$, given $D$.

likelihood of the random hypothesis is 174 times greater than that of the simultaneous hypothesis. The situation for G-7 is ambiguous: $P(D{:}H_r)\,/\,P(D{:}H_s) = 1.8$.

In summary, on the basis of individual correlations between pairs of flows using the paleomagnetic evidence, each of six pairs might well be outcrops of the same lava flow, one is almost certainly not, and one is ambiguous. However, in only two of the cases is the probability of $H_s$ compared to $H_r$ greater than 19:1 (95 % confidence). Thus we need to demonstrate at a higher level of probability that the two sections are temporally equivalent, even though some of the flows may not be the same.

*Correlations between sequences of flows.* As can be seen in Table 1, it is really the sequence of paleomagnetic directions--from shallow to moderate to steep--that ties the two sections of flows to each other. We would like to calculate the relative probabilities of the random and simultaneous hypotheses for the two sequences of flows. A convenient way to do this is to apply Bayes theorem (1*a*) sequentially. This is illustrated on the right side of Table 2. We start with flows Rocky Coulee and 11 and assign prior probabilities $P(H_r) = P(H_s) = 0.5$. The posterior probabilities calculated from (1*a*) serve as the prior probabilities for the next lower pair of flows in the section, and so on. We find the end result at the bottom of Table 2: $P(H_s{:}D) > 0.999$ and $P(H_r{:}D) = 9\times10^{-6}$. In other words, the consistency of the paleomagnetic data between the eight pairs of flows is explained at least $10^5$ times more probably by the simultaneous hypothesis than by the random hypothesis.

*Correlations between groups of flows.* We have already argued from individual correlations that at least one and probably two of the pairs of flows were not erupted simultaneously. These pairs both belong to the groups of flows with moderate inclinations. At both Sentinel Gap and Umtanum Ridge the moderate inclinations span a 17° range, but the pattern of inclination variation is not the same. Some of this discrepancy can be attributed to errors but not all. Thus, despite the qualitative appearance of the data, we cannot accept the hypothesis that the flows with moderate inclination were erupted within the same geomagnetic instant (and therefore drawn from populations with the same mean): it fails the F test [*Watson*, 1956] at greater than 99 % probability at both sites.

An alternative and more realistic hypothesis than any discussed so far is that the flows with moderate inclinations were erupted during a fairly brief episode of volcanism but that at least one of these flows in each section does not have an exact temporal equivalent (in the geomagnetic sense) in the other. In this case we find the mean direction for each group of six flows and ask whether the consistency of the means is best explained by the random or the simultaneous hypothesis. The random error is now taken to be the angular standard deviation of the mean for each group: $\langle \bar{s}_g \rangle = 2.9°$ and 2.6°, respectively, for Sentinel Gap and Umtanum Ridge. This is larger by 1°-2° than the average of $M = 6$ within flow dispersions of the means

$$\langle \bar{s}_w \rangle = \frac{1}{M}\left[\sum_{i=1}^{M} \bar{s}_{w,i}^{\,2}\right]^{1/2} \qquad (13)$$

Presumably, this is because some additional scatter between flows is caused by real changes in geomagnetic field directions during the volcanic episode, variations in original dip between the flows, and perhaps other causes.

The tectonic error of the group mean will be less than or equal to the average tectonic error of the flow means, depending on what proportion of it varies randomly from flow to flow. If the tectonic error was constant and in the same direction for all the flows in both sections, we would expect to see a systematic difference in direction between corresponding flows or groups of flows. There is a tendency toward more shallow inclinations at Sentinel Gap, but it is not very pronounced. We will somewhat arbitrarily assume that half the tectonic variance is expressed as between flow scatter and half affects the group as a whole. This results in an estimate of $<\bar{s}_t> = 2.5°-2.6°$ for the sections. The systematic error due to field anomalies is assumed to be the same as for individual lava flows: $<\bar{s}_f> = 0.5°$. This is a maximum estimate, but even so it is so small as to have a negligible effect on the calculation.

The group mean directions are rather close to each other ($\alpha = 2.8°$). With the above estimates of error we calculate $P(D:H_s) = 0.762$ from (7). The likelihood associated with the random hypothesis is much smaller, even though both group-mean directions lie within 2° of the axial dipole direction: $P(D:H_r) = 0.035$. Thus the simultaneous hypothesis is almost 22 times more likely than the random hypothesis. By comparison, applying Bayes theorem sequentially to the six pairs of flows yields $P(H_s:D) = 0.929$ and $P(H_r:D) = 0.071$, or $H_s$ is about 13 times as likely as $H_r$. These results are comparable, but the former is preferred because in the latter case the relative probabilities fluctuate wildly as the calculation progresses from pair to pair.

We can combine the likelihoods for the moderate groups with those for Rocky Coulee-11 and D-4 (Table 2) by applying Bayes theorem to the three comparisons in turn. The end result is that $P(H_s:D)$ is more than $10^5$ times greater than $P(H_r:D)$. Hence we conclude that the sequence of shallow, moderate and steep paleomagnetic directions encountered at both sections implies with a very high degree of probability that the sections are stratigraphically equivalent. With only paleomagnetic information it is not possible to prove definitively whether outcrops at two sites belong to the same flow. We can say that the paleomagnetic consistency between individual flows allows the hypothesis that eight or nine of the 10 flows in the two sections are the same. Considering the proximity of Sentinel Gap and Umtanum Ridge and the thickness of the flows, this result is not too surprising.

### *Sentinel Gap-Butler Canyon*

A geologically more interesting example involves a sequence of flows sampled by *Watkins and Baksi* [1974] at Butler Canyon located 180 km to the SW in Oregon (Figure 3). Having obtained a K-Ar age of 14.6 ± 0.3 m.y. from a normally magnetized flow, Watkins and Baksi placed the flows in the Grande Ronde Basalt Formation. *Nathan and Fruchter* [1974] showed that the section at Butler Canyon (their Tygh Ridge section) is of the Grande Ronde chemical type and is capped by a flow of the Frenchman Springs chemical type. This evidence suggests that the flows at Butler Canyon are at the top of the Grande Ronde section and thus stratigraphically equivalent to the flows at Sentinel Gap.

The sequence of flows 10 through 2 at Butler Canyon (Table 1) are plotted on an equal area net in Figure 4. As was the case at Sentinel Gap, the directions are clustered and suggest that volcanism was episodic. Moreover, the Butler Canyon flows have recorded approximately the same three field directions, in sequence, which were found at Sentinel Gap. Both sites have pairs of flows with unusually steep and unusually shallow westerly directions, separated by a number of flows with moderate directions close to the expected axial dipole direction. There is one more of these flows with moderate direction at Butler Canyon than at Sentinel Gap.

It is more difficult to estimate the errors at Butler Canyon. *Watkins and Baksi* [1974] collected only two samples per flow. This has the effect of increasing both the within-flow standard deviation of the mean and its uncertainty. In addition, they did not discuss tectonic corrections applied to their results, nor have we visited the Butler Canyon section. Since our estimate of tectonic error at Sentinel Gap varies from 3.5° to 4.5°, we have arbitrarily chosen a value of $\bar{s}_t = 4.0$ for the flows at Butler Canyon. This may well be an underestimate of the error, which would lead to a conservative evaluation of our confidence in the stratigraphic correlation. Finally because of the much greater separation between sections, we estimate a larger standard deviation of the mean due to field anomalies than in the previous example; $\bar{s}_f = 1.0°$. This allows for greater differences in the nondipole field and in the magnetic basement at the more distant Butler Canyon section.

The errors estimated for the Butler Canyon flows and the equivalent $\bar{k}$'s are given in Table 1. In addition, the flow mean directions should be steepened by a 1.2° rotation about an E-W axis to compensate for the difference in axial dipole field direction between the two sections. In other cases where the localities being compared are much farther apart than Butler Canyon and Sentinel Gap, it would be preferable to transform the field directions to VGP's. However, it is simpler to use the field directions when only a minor correction for latitudinal separation is necessary.

*Correlation between groups of flows.* Flow-by-flow correlation as was done for the Sentinel Gap-Umtanum Ridge comparison is not practical in this case. One difficulty is that the Butler Canyon group with moderate directions has one less flow than the moderate group at Sentinel Gap. Another is the lack of information concerning tectonic corrections at Butler Canyon. A third is that the number of samples collected per flow at Butler Canyon ($N = 2$) was probably not enough to obtain a sufficiently accurate estimation of the mean and its uncertainty. These difficulties are lessened considerably however, by making comparisons between groups of flows.

We assume that at Butler Canyon the tectonic error for each group is 4.0°, the same as for each flow. At Sentinel Gap, as before, $<\bar{s}_t>$ is reduced somewhat below the average value of $\bar{s}_t$ for the flows. The error due to field anomalies is assumed to be the same as for individual flows. Finally, the between-flow standard deviation of the group mean, $<\bar{s}_g>$, is taken as the estimate of the random error except for the shallowly inclined group at Butler Canyon. For this group we use the average within-flow estimate of the uncertainty, $<\bar{s}_w>$, which is greater than $<\bar{s}_g>$ and is considered to be more realistic.

These estimates of error, other relevant parameters, and the results of the probability calculations are given in Table 3. For the shallowly magnetized flows, the probability of the simultaneous hypothesis is almost 200 times greater than the probability of the random hypothesis. This result reflects the fact that the average shallow directions from Sentinel Gap and Butler Canyon are rather close to each other and quite far from the axial dipole direction, and that the overall uncertainty $<\bar{s}>$ is fairly large.

For the group of flows with moderate directions, the correlation suggested by the paleomagnetism is not as strong. Still, $H_s$ is about 5 times as likely as $H_r$, even though we have been conservative in estimating uncertainties for $P(D:H_s)$ and generous in estimating paleosecular

TABLE 3. Sentinel Gap-Butler Canyon Correlation (Group by Group)

| Flows | $\langle \bar{s}_g \rangle$ | $\langle \bar{s}_w \rangle$ | $\langle \bar{s}_t \rangle$ | $\langle \bar{s}_f \rangle$ | $\langle \bar{s} \rangle$ | $\langle \bar{k} \rangle$ | $\delta$ | $\alpha$ | $P(D:H_r)$ | $P(D:H_s)$ | Sequential Application of Bayes Theorem: $P(H_r)$ | $P(H_s)$ | $P(H_r:D)$ | $P(H_s:D)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| RC-MUS | 2.5 | 1.1 | 2.5 | 0.5 | 3.6 | 507 | 14.7 | 8.3 | 0.095 | 0.179 | 0.500 | 0.500 | 0.347 | 0.653 |
| 3-4 | 3.1 | 2.6 | 4.0 | 1.0 | 5.2 | 243 | 18.5 | | | | | | | |
| E-J | 2.9 | 0.7 | 2.6 | 0.5 | 3.9 | 432 | 1.9 | 5.0 | 0.100 | 0.494 | 0.347 | 0.653 | 0.097 | 0.903 |
| 8-4 | 1.8 | 1.4 | 4.0 | 1.0 | 4.5 | 324 | 5.9 | | | | | | | |
| C-D | 5.3 | 3.3 | 3.2 | 0.5 | 6.2 | 171 | 27.2 | 5.5 | 0.004 | 0.763 | 0.097 | 0.903 | 0.001 | 0.999 |
| 10-9 | 3.8 | 7.6 | 4.0 | 1.0 | 8.6 | 89 | 30.9 | | | | | | | |

The $\langle \bar{s}_g \rangle$, $\langle \bar{s}_w \rangle$, $\langle \bar{s}_t \rangle$, and $\langle \bar{s}_f \rangle$ are the angular standard deviation of a group mean direction due to the dispersion of flow mean directions, average within-site dispersion of the grouped flows, improper correction for tilts of the flows and magnetic field anomalies; $\langle \bar{s} \rangle$ is the total estimated angular standard deviation of the mean direction found by combining $\langle \bar{s}_t \rangle$, $\langle \bar{s}_f \rangle$, and the larger of $\langle \bar{s}_g \rangle$ and $\langle \bar{s}_w \rangle$; $\langle \bar{k} \rangle$ is the precision parameter equivalent to $\langle \bar{s} \rangle$. Other symbols as in Table 2.

variation. The moderately magnetized flows are less useful as stratigraphic markers simply because they have directions which are closer to the axial dipole direction and hence are not as distinctive.

The correlation is weakest for the steeply magnetized groups. $H_s$ is only about twice as likely as $H_r$, even though these directions are relatively unusual. The reason is that the mean directions are more than 8° apart, despite the fact that the flows all appear to be good paleomagnetic recorders. It would be interesting to see if this discrepancy would diminish with more detailed sampling at Butler Canyon.

As was the case with the Umtanum Ridge-Sentinel Gap correlation, however, it is really the sequence of ancient field directions–shallow to moderate to steep–which suggests most strongly that the flows at Sentinel Gap and Butler Canyon are so closely related in time. It is highly improbable that flows at the two sites could have erupted at significantly different times, yet possess such similar sequences of paleomagnetic directions. Combining the results for all three flow groups (see Table 3), we find that the probability of $H_s$ is approximately 1000 times greater than the probability of $H_r$. Thus, even though the data available for Butler Canyon are not as well controlled as we would like for making correlations based on paleosecular variation, we still are able to conclude that the two sections are very probably stratigraphically equivalent.

*Discussion.* By stratigraphic equivalence we do not necessarily mean flow-for-flow equivalence. The observed correlation could arise from nearly simultaneous eruptions from sources near each of the sites. This eruptive pattern might be expected along linear vent systems like those described for younger Columbia River flows by *Swanson et al* [1975]. However, the extent of observed vent systems is not great enough to explain sites as distant as Sentinel Gap and Butler Canyon. Furthermore, no Grande Ronde dikes have yet been discovered near Sentinel Gap or Butler Canyon. Thus it is simplest to hypothesize that many of the same flows are exposed at the two sections. This would be an interesting finding, since individual Grande Ronde flows usually cannot be traced in the field for more than a few tens of kilometers [*Swanson et al.*, 1979]. If there are indeed no source vents hidden below the basalts in central Washington, then our results suggest that many Grande Ronde flows have lateral extents of at least several hundred kilometers.

## Conclusions

The variation in paleomagnetic direction due to secular variation can be a useful tool for stratigraphic correlation. We have developed a statistical method for evaluating such correlations between pairs of sites. It involves comparing the probability that the similarity of the paleomagnetic directions at the two sites could arise by chance with the probability that the difference between the two directions can be attributed to errors in the paleomagnetic recording, sampling, and measuring process. To calculate the former probability, one must estimate the dispersion of field directions characteristic of secular variation at the time and place appropriate to the rock units being correlated. To calculate the latter probability, one must estimate the magnitudes of the errors. In our study the most important error was usually tectonic, and it was usually also the most difficult to assess. In studies of this sort more time and effort needs to be devoted to determining tectonic correction and its uncertainty.

We applied the method to three sections, each comprising 8-10 lava flows, of the Columbia River Basalt Group. For both pairwise comparisons the analysis revealed that simultaneous magnetization of the flows was a much more likely explanation of the paleomagnetic results than chance agreement of ancient field directions. The simplest way to explain this simultaneity is to suppose that many of the same flows are exposed at the three sites. This conclusion was expected for the Sentinel Gap-Umtanum Ridge comparison, but for Sentinel Gap and Butler Canyon the result is surprising and, if true, has interesting implications. Since Sentinel Gap and Butler Canyon are 180 km apart, our conclusion implies that series of Grande Ronde flows extended over distances of several hundred kilometers. Detailed paleomagnetic correlation of other sites in the Columbia River basalt should shed further light on this interesting problem.

## Appendix A. Graphs To Estimate $P(D:H_r)$

The curves shown in Figure A1 give the probability associated with all directions lying within $\alpha$ (degrees ) of a reference direction, which itself is $\delta$ (degrees ) from the center of the Fisher probability distribution. We have drawn these curves for six values of the precision parameter

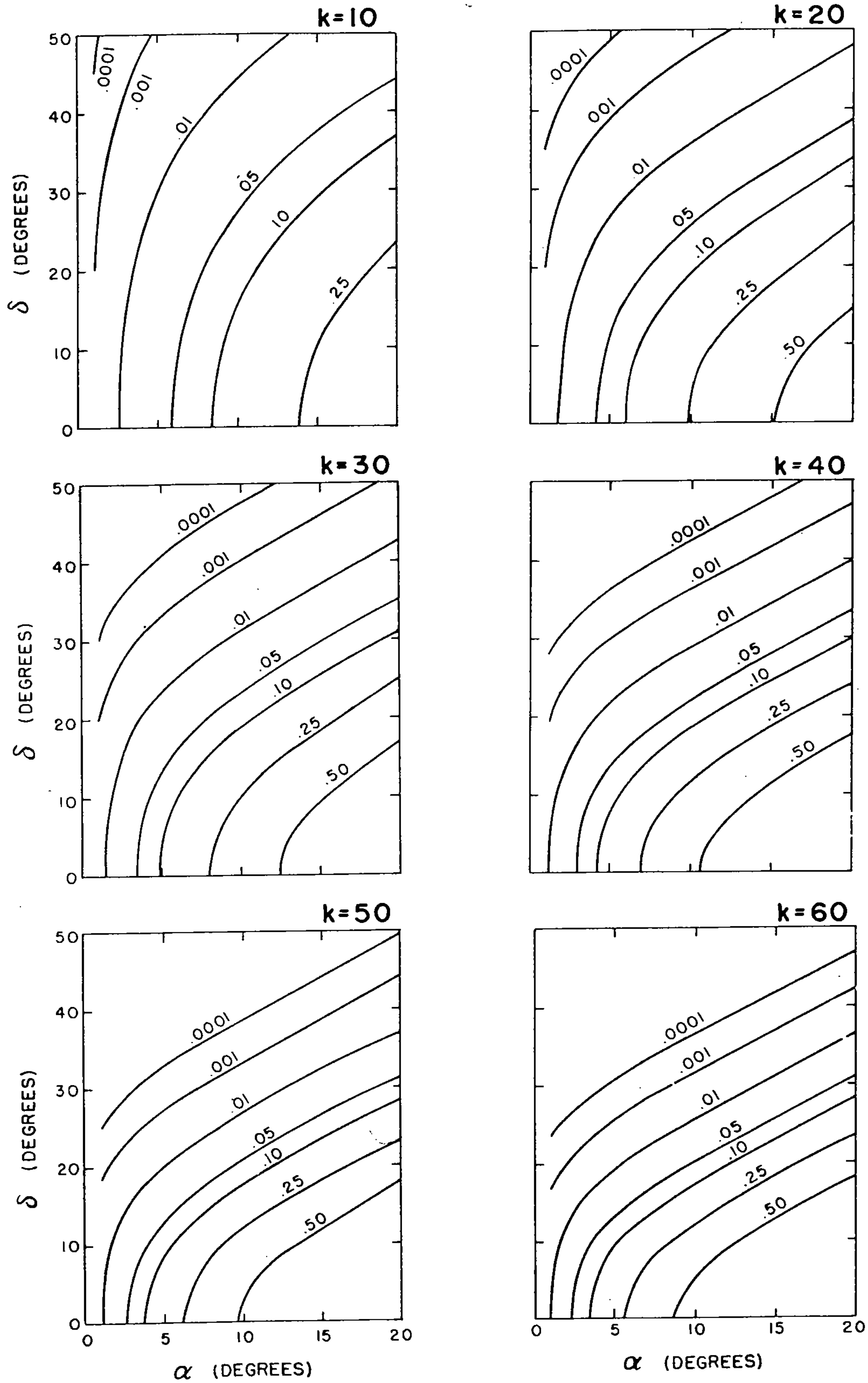


Fig. A1. Graphs to estimate $P(D:H_r)$; $\alpha$ and $\delta$ defined as in Figure 1.

$k$ which span the range expected for paleosecular variation.

These curves can be used to estimate the likelihood $P(D:H_r)$ of the random hypothesis for directions $\vec{X}$ and $\vec{Y}$. For example, assume that the paleosecular variation is described by $k = 40$, that $\vec{X}$ lies 10° from the axial dipole direction, and that $\vec{Y}$ lies 14° from the axial dipole direction and 5° from $\vec{X}$. The probability associated with all directions lying as close to $\vec{X}$ as $\vec{Y}$ ($\alpha = 5°$, $\delta = 10°$) may be read off the graph for $k = 40$, and is approximately 0.07. The probability of all directions as close to $\vec{Y}$ as $\vec{X}$ ($\alpha = 5°$,

$\delta$ = 14°) is close to 0.05. Averaging these values gives 0.06, the likelihood $P(D{:}H_r)$ of sampling the ancient field at two different times, and obtaining directions as similar to one another, and as far from the axial dipole direction, as $\vec{X}$ and $\vec{Y}$. By interpolating between the curves presented here, $P(D{:}H_r)$ for any value of $k$ between 10 and 60 may be found.

## Appendix B. Comparison Of $P_S$ With $P_F$

For the special case of two populations of unit vectors with the same precision parameter, we can check the value of $P(D{:}H_s)$ which we determine by (7) (and will abbreviate for the following discussion as $P_S$) versus the probability ($P_F$) derived from the F test [*Watson*, 1956]. We suppose that $N_1$ and $N_2$ samples, drawn from two different populations with the same precision parameter $\kappa$, yield mean directions $\alpha$ degrees apart. *Watson* [1956] showed that

$$\frac{(N-2)(R_1+R_2-R)}{(N-R_1-R_2)} = F_{2,2(N-2)} \qquad \text{(B1)}$$

where $N = N_1 + N_2$, $R_1(R_2)$ is the magnitude of the resultant of the $N_1(N_2)$ unit vectors drawn from population 1(2), $R$ is the magnitude of the resultant of all N unit vectors, and F is Fisher's F-statistic (with 2 and 2($N$-2) degrees of freedom) which is found in most standard textbooks on statistics. In a manner analogous to the examples of *McFadden* [1980, Appendices A and B], one can show that (B1) is an excellent approximation for most sample collections of interest in paleomagnetism, subject, of course, to the severe limitation that $\kappa_1 = \kappa_2$.

When the left hand side of (B1) is large, the probability is small that the populations have the same true mean. The probability $P_F$ that a particular value $F_{2,2(N-2)}$ given by (B1) will be exceeded is

$$P_F = \left[1 + \frac{F_{2,2(N-2)}}{N-2}\right]^{-(N-2)} \qquad \text{(B2)}$$

Given $N_1$, $N_2$, $R_1$, $R_2$ and $\alpha$, and using the relation

$$R = (R_1^{\,2} + 2R_1R_2\cos\alpha + R_2^{\,2})^{1/2} \qquad \text{(B3)}$$

we can find $P_F$ from (B1) and (B2). Since the precision parameters are assumed the same, we also have

$$k_w = \frac{N_1 - 1}{N_1 - R_1} \text{ or } \frac{N_2 - 1}{N_2 - R_2} \qquad \text{(B4)}$$

as an estimate of $\kappa$. Then we can calculate the corresponding probability $P_S$ by our method using (7) with

$$\bar{k}_X = N_1 k_w \qquad \text{(B5)}$$

$$\bar{k}_Y = N_2 k_w \qquad \text{(B6)}$$

$$\beta_X = \frac{N_2\alpha}{N_1+N_2} \qquad \text{(B7)}$$

$$\beta_Y = \frac{N_1\alpha}{N_1+N_2} \qquad \text{(B8)}$$

The results of the comparison of $P_F$ and $P_S$ are shown in Table B1. $P_F$ is always larger than $P_S$, but the difference between them goes to zero for probabilities approaching 1 and 0. The maximum difference between $P_F$ and $P_S$ occurs in the probability range 0.1 to 0.35, but the greatest relative difference occurs in the limit as $P_F$ approaches zero. For $N_1 = N_2 = 7$, the maximum of ($P_F - P_S$) varies from 0.022 with $k_w$ = 500 to 0.084 with $k_w$ = 5. Table B2 illustrates how ($P_F - P_S$) increases with decreasing N.

Our calculation always underestimates the probability which means once again that our estimate of confidence in a correlation is too conservative. The errors are not great. For the range of parameters in Table 2 the average difference between $P_F$ and $P_S$ is 0.01. The only significant error is in the F-6 comparison, where there is a large relative error because $P_S$ is very low (0.003). Applying (B1) and (B2), which is not strictly valid since $k_{w,1}$ is significantly different from $k_{w,2}$ we obtain $P_F$ = 0.010. This does not raise our confidence particularly in the F-6 correlation, because the probability is still extremely low, nor does it significantly increase our confidence in the correlation of the whole sequence, which was already very high.

In Table 3 we are dealing with values of $N$ as small as 4. Thus for a given $k_w$ the difference between $P_F$ and $P_S$ will be greater than in Table B1. Using equations (B1) and (B2), we find ($P_F - P_S$) is only 0.008 for the shallow flows and 0.014 for the moderate flows. On the other hand, for the steep flows $P_F$ = 0.285, which is greater than $P_S$ by 0.111. Using the larger value for $P(D{:}H_s)$ would increase the likelihood of $H_s$ from 2 times as likely as $H_r$ to 3 times as likely as $H_r$. Again, this has no significant effect on our interpretation.

Finally, we should mention a subtle point which shows that equation B1 is not strictly applicable to our problem, even when there is no significant difference in precision parameter between the populations being compared. In *Watson*'s [1956] F test, the directions of the samples are given and the precision parameter, having cancelled as a common factor in the numerator and denominator of (B1), is never needed. In our problem, however, the precision parameter enters explicitly because a large fraction of the uncertainty in direction that it represents arises from systematic effects such as tectonic movements and magnetic anomalies which are estimated independently from the sample-to-sample dispersion. That is to say, our value of $\bar{k}$ is a hybrid quantity reflecting errors estimated from the

TABLE B1. Values of ($P_F - P_S$) as a Function of $k_w$ and $P_F$ for $N_1 = N_2 = 7$

| | $P_F$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k_w$ | 0.95 | 0.80 | 0.65 | 0.50 | 0.35 | 0.20 | 0.05 | 0.01 | 0 |
| 500 | <0.001 | 0.003 | 0.006 | 0.010 | 0.017 | 0.022 | 0.017 | 0.010 | 0 |
| 100 | 0.001 | 0.003 | 0.008 | 0.013 | 0.019 | 0.024 | 0.018 | 0.011 | 0 |
| 50 | 0.001 | 0.005 | 0.010 | 0.016 | 0.022 | 0.027 | 0.019 | 0.011 | 0 |
| 20 | 0.002 | 0.010 | 0.018 | 0.025 | 0.032 | 0.035 | 0.021 | 0.012 | 0 |
| 10 | 0.004 | 0.018 | 0.031 | 0.042 | 0.049 | 0.048 | 0.026 | 0.013 | 0 |
| 5 | 0.010 | 0.038 | 0.061 | 0.077 | 0.084 | 0.078 | 0.034 | 0.016 | 0 |

TABLE B2. Values of $(P_F - P_S)$ as a Function of $k_w$ and $N$ for $P_F = 0.25$ and $N_1 = N_2$

| | N | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $k_w$ | 4 | 6 | 8 | 14 | 20 | 40 | 200 | 2000 |
| 500 | 0.115 | 0.060 | 0.040 | 0.021 | 0.014 | 0.007 | 0.002 | 0.001 |
| 50 | 0.117 | 0.063 | 0.045 | 0.026 | 0.020 | 0.013 | 0.012 | 0.012 |
| 5 | 0.142 | 0.102 | 0.090 | 0.081 | 0.077 | 0.075 | 0.073 | 0.073 |

directional data and from other sources. Thus, to apply (B1), we have to compute values of $R_1$ and $R_2$ which are consistent with $\bar{k}$ using

$$R_1 = N - \frac{N(N-1)}{\bar{k}_1} \quad \text{(B9)}$$

and similarly for $R_2$. Thus the values of $R_1$ and $R_2$ we use are estimates, not directly observed quantities, and we cannot expect (B1) and (B2) to give perfectly correct probabilities.

## Notation

A capitalized subscript associates a parameter with a particular flow. A bar over a statistical parameter indicates that the parameter refers to flow mean rather than sample directions. We use brackets when treating a sequence of flows as a group.

### Probability and directions

- $H_r$ 'random' hypothesis.
- $H_s$ 'simultaneous' hypothesis.
- $D$ paleomagnetic data.
- $P(A)$ unconditional probability of A.
- $P(A:B)$ conditional probability of A; that is, the probability of A given B.
- $P_{XY}$ probability of flow Y having a paleomagnetic direction similar to flow X.
- $P_X$ probability of flow X having a paleomagnetic direction dissimilar to ancient field direction.
- $P_X' = 1 - P_X$
- $N, N_X$ number of sample directions.
- $M$ number of flow mean directions included in group mean.
- $R, R_X$ length of vector resultant of sample or flow-mean directions.
- $\alpha_{95}, \alpha_{95X}$ 95% confidence interval about mean direction.
- $\alpha, \alpha_X$ angular distance between two paleomagnetic directions.
- $\beta, \beta_X$ angular distance between paleomagnetic and ancient field directions.
- $\delta, \delta_X$ angular distance between paleomagnetic and axial dipole directions.

### Estimates of $\kappa$ (Fisher precision parameter)

- $k$ precision parameter describing dispersion of geomagnetic field directions due to secular variation.
- $s$ angular standard deviation equivalent to $k$.
- $k_w, k_{wX}$ precision parameter describing within-site dispersion.
- $s_w, s_{wX}$ angular standard deviation equivalent to $k_w$.
- $\bar{k}_w, \bar{k}_{wX}$ precision parameter describing dispersion of flow-mean due to within-site dispersion.
- $\bar{s}_w, \bar{s}_{wX}$ angular standard deviation equivalent to $\bar{k}_w$.
- $\bar{s}_t$ angular error in flow-mean direction due to original tilt and tectonic correction.
- $\bar{s}_f$ angular error in flow-mean direction due to field anomalies.
- $\bar{s}, \bar{s}_X$ total angular error for flow-mean direction.
- $\bar{k}, \bar{k}_X$ precision parameter equivalent to $\bar{s}$.
- $\langle\bar{s}_g\rangle$ angular standard deviation of group mean due to dispersion of flow-mean directions.
- $\langle\bar{s}_w\rangle$ angular standard deviation of group mean due to within-site dispersion of each flow mean.
- $\langle\bar{s}_t\rangle$, $\langle\bar{s}_f\rangle$, $\langle\bar{s}\rangle$, $\langle\bar{k}\rangle$ as above, except referred to group mean

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